

Application of a Data Driven Method to Isotopes Identification in the AMS experiment

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November 1, 2022

Abstract. The identification of isotopes in AMS (Alpha Magnetic Spectrometer) is of paramount importance to deepen our understanding of cosmic ray propagation. The AMS, which has been installed on the International Space Station since 2011, is able to measure different properties of incoming particles like velocity, rigidity and charge. Velocity and rigidity, or the combined variable mass, are the main ingredients to isotopic separation. This requires building isotopic mass templates which combined will provide isotopic abundance. In this work, a data-driven method will be applied to Monte Carlo (MC) data samples to extract mass templates for cosmic-ray protons and deuterons. Additionally, we will show that we were able to retrieve the simulated abundances.

KEYWORDS: Isotopes Identification, AMS, Data Driven Method

1 Introduction

1.1 The AMS detector

The Alpha Magnetic Spectrometer (AMS) is a precision particle physics detector mounted on the International Space Station (ISS), whose main objectives are the search for anti-matter, understanding the origin of dark matter and measuring cosmic rays fluxes. Several constituents and their functions are represented in figure 1, which is taken from [1], the latest report from the AMS collaboration, where they are described in detail.

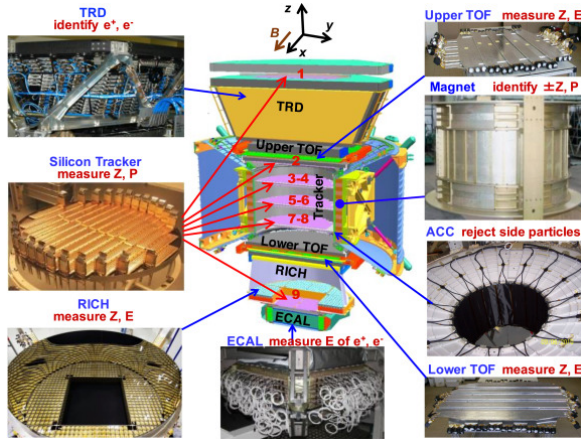


Figure 1: Overview of AMS detectors.

For the present work, related to isotopic identification, the most relevant are silicon Tracker, RICH, TOF and the magnet.

1.2 Isotopes Identification

The ability to identify isotopes in the AMS is important and relevant for studies of cosmic ray propagation processes, for example, to characterize the structure of the magnetic fields in the galactic halo [2].

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Physical Observables

To identify the isotopes crossing the detectors three physical observables are taken into account: the charge of the particle, Z ; its velocity¹, β ; and its rigidity² $R = \frac{pc}{eZ}$. Once we have these measurements we can calculate the mass, $m = \frac{ReZ}{\gamma\beta}$, of the particle and thus identify the isotope.

Detectors

To make measurements of these observables three detectors are relevant:

- **Silicon Tracker** - consists of a nine layer silicon sensor detector responsible, together with the magnet, for determining the trajectory and charge of cosmic rays and their loss of energy [1]. The characterization of the trajectory of the particles, including the measurement of the sagitta, enables us to assign a rigidity value to incoming particles.
- **Time of Flight (ToF)** - consist of a set of 4 scintillator planes and a coincidence unit that provides us with a charged particle trigger and with a measurement of the velocity of cosmic rays.[1]
- **Ring Imaging Cherenkov Detector (RICH)** - consists of two radiating media: 16 tiles of sodium fluoride, NaF, and 92 tiles of silica aerogel. It exploits the phenomenon of Cherenkov radiation to provide us with a measurement of passing particles' velocity and charge. It can measure velocities $\beta_{NaF} > 0.75$ and $\beta_{aerogel} > 0.953$. [1]

1.3 Overview of the article

This article is aiming at develop and implement a mass separation method able to evaluate the statistics of every isotope present in a data sample. With that in mind we

¹ $\beta = \frac{v}{c}$, where c is the speed of light

²it is a measurement of how much the particle is deflected by a magnetic field

first created a MC sample of events, taking into account the AMS detector's characteristics.

Second, we describe an iterative data-driven method from [2] and implement it to try to extract the isotopes' mass templates, directly from MC data.

Third, a solution is offered when a problem with determining the fractions of isotopes has been identified. We demonstrate that it is possible to determine the right isotope fraction in our simulated mass distributions and to determine the templates of the isotopes.

2 Isotopes Simulation

The data used for the analysis described in the following sections has been simulated. This simulation is divided in two steps: first, we generate the true properties of the particle; second, the measurement on the detectors. The ultimate objective of these simulations was to create mass histograms to which the data-driven method will be applied.

2.1 Particle Generation

We start by generating a true kinetic energy per nucleon, $T_n = \frac{T}{A}$, a quantity which is closely related to β . We want to pick values which follow the differential-energy spectrum of observed cosmic rays, which can be modeled using the force-field approach [3]. This approach adds a correction to the unmodulated local interstellar spectrum (LIS) to account for the influence of the heliosphere. Assuming an unmodulated LIS, [3],

$$J_{LIS}(T_n) = \left(\frac{1.9 \times 10^4}{m^2 \text{ sr s GeV/n}} \right) \frac{P(T_n)^{-2.78}}{1 + 0.4866 \cdot P(T_n)^{-2.51}} \quad (1)$$

where $P(T_n) = \sqrt{T_n(T_n + 2m_n)}$ and $m_n = 0.938 \text{ GeV/n}$ is the nucleon mass. The kinetic energy per nucleon is given in GeV/nucleon so that J_{LIS} is expressed in particles/($m^2 \cdot \text{sr} \cdot \text{s} \cdot \text{GeV/nucleon}$). The energy spectrum of the isotope (with charge Z and mass number A) is obtained by correcting this unmodulated LIS with [3]:

$$J(T_n, \varphi) = J_{LIS}(T_n + \Phi) \frac{(T_n)(T_n + 2m_n)}{(T_n + \Phi)(T_n + \Phi + 2m_n)} \quad (2)$$

where $\Phi = (eZ/A)\varphi$, being φ the modulation potential [GV], a time-varying function related to solar activity. From expression (2) we sample values of T_n . For a given sample we can calculate the true β and rigidity according to the relations:

$$\beta(T_n) = \frac{\sqrt{T_n(T_n + 2m_n)}}{T_n + m_n} \quad (3)$$

$$R(T_n) = \frac{A}{Z} \sqrt{T_n(T_n + 2m_n)} \quad (4)$$

2.2 Measurement Simulation

Rigidity

To simulate the rigidity measurement we assumed that its relative precision is roughly 10%. We generate the measured quantity, R_m , from a normal rigidity distribution:

$$p(R_m|R) = \frac{1}{\sigma_R \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left(\frac{R_m - R}{\sigma_R} \right)^2 \right\} \quad (5)$$

where the mean value, R , is the true rigidity and $\sigma_R = 0.1 R$.

Velocity

To simulate the velocity measurements a similar approach was used. In this case we have three different measurements to account for, one from the TOF and two from the RICH. The relative precision of TOF measurement is [4]:

$$\left(\frac{\sigma_\beta}{\beta} \right)_{TOF} = \beta \cdot c [\text{cm/ps}] \cdot \frac{\sqrt{\left(\frac{159 [\text{ps}]}{Z} \right)^2 + (79 [\text{ps}])^2}}{120 \sqrt{2} [\text{cm}]} \quad (6)$$

For the RICH measurement some considerations related to the acceptance need to be made: it was assumed that roughly 80% of particles in AMS cross the RICH detector radiator plane and from these 11% of AMS particles cross NaF and the remaining (89%) cross the aerogel. The velocity resolutions are:

$$\left(\frac{\sigma_\beta}{\beta} \right)_{NaF} = \frac{0.34\%}{0.8Z} \quad (7)$$

$$\left(\frac{\sigma_\beta}{\beta} \right)_{AGL} = \frac{0.12\%}{0.8Z} \quad (8)$$

As for rigidity, the measured quantity, β_m was sampled according to a normal distribution.

Mass Distributions

In figure 2, below, we can see the results of a simulation of protons and deuterons ($A=1$ and $A=2$), where the relative real fractions were set to be $\{0.5, 0.5\}$. There are two graphs, one relative to TOF and the other to RICH, where we plot measured rigidity on the x axis and measured velocity on the y axis; the number of events is represented in the z axis.

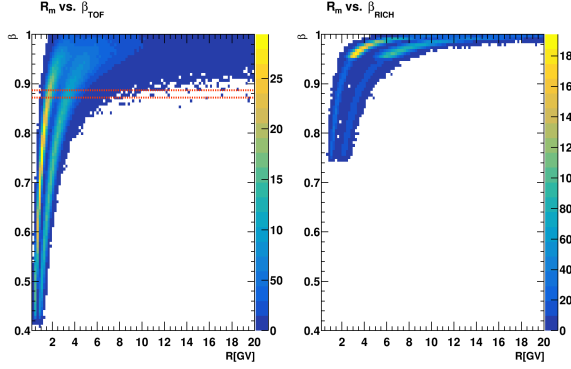


Figure 2: Measured β and rigidity.

To build mass distributions, events are selected in small β_m intervals and the mass of the particle is calculated (from β_m and R_m),

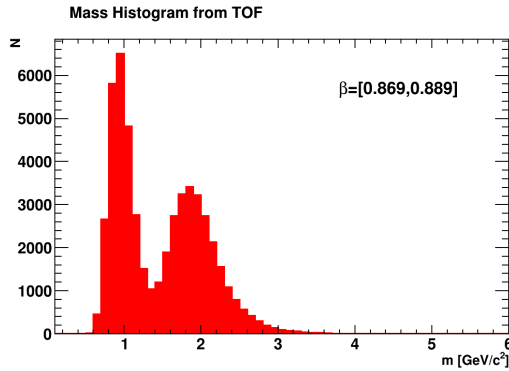


Figure 3: Mass histogram.

We alter the variables in this way to facilitate interpretation.

3 Data Driven Method

The data driven method (DDM) is an iterative tool that can be applied to a given (total) distribution which is the sum of several unknown elementary distributions in order to find each elementary distribution and its relative fraction. This method will be applied to isotopes with charge ($Z=1$) identification.

3.1 Problem Statement and Solutions

The problem is the following: a given mass histogram is provided, like the one in figure 3, and a set of isotopes of a given charge, Z , is known to be present; we want to know the mass distributions of each isotope, which will be called mass templates, and the relative fractions of each isotope present. Several approaches (analytical, parametric [5], data-driven) can be used to find the solutions. The data-driven approach will be explored.

3.2 Assumptions

The DDM that will be later described is based on two assumptions:

1. The mass resolutions are equal for all the isotopes
2. It is possible to transform one mass template into another with a linear transformation

Equal Mass Resolutions

The mass resolution is given by:

$$\frac{\delta M}{M} = \sqrt{\left(\frac{\delta R}{R}\right)^2 + \gamma^4 \left(\frac{\delta \beta}{\beta}\right)^2} \quad (9)$$

We can assume that the β resolution is nearly constant because the mass histograms are constructed from small β bins. The rigidity resolution is also constant in the low kinetic energy range [6]. If both β and rigidity resolutions are constant so is the mass resolutions. If the different isotopes (α , λ and θ) have similar masses we can assume equal mass resolutions for them.

$$\left(\frac{\delta M}{M}\right)_\alpha = \left(\frac{\delta M}{M}\right)_\lambda = \left(\frac{\delta M}{M}\right)_\theta \quad (10)$$

Transforming Templates

The claim is that we can get the mass template of isotope β by applying a linear transformation to the α isotope template.

$$T_\beta(x) = \mathcal{L}_{\alpha,\beta} T_\alpha(x) \quad (11)$$

where x is the mass variable and $\mathcal{L}_{\alpha,\beta}$ represents the transformation from α to β . This transformation could have the general form:

$$\mathcal{L}_{\alpha,\beta} T_\alpha(x) \equiv k T_\alpha(ax + b) \quad (12)$$

We start restricting this family of functions by imposing that after the transformation the resulting function must be normalized (in order to be a mass distribution function):

$$\int_{-\infty}^{\infty} k T_\alpha(ax + b) dx = 1 \quad (13)$$

Performing the substitution $y = ax + b$:

$$\int_{-\infty}^{\infty} k T_\alpha(y) \frac{dy}{a} = \frac{k}{a} \int_{-\infty}^{\infty} T_\alpha(y) dy = 1 \quad (14)$$

From this equality we conclude that k must be equal to a since T_α was normalized.

If the templates are gaussian functions with mean values μ_α , μ_β and standard deviation σ_α and σ_β we can restrict the values of a and b after equating:

$$T_\beta(x) = \mathcal{L}_{\alpha,\beta} T_\alpha(x) = \frac{1}{\sigma_\beta \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu_\beta}{\sigma_\beta} \right)^2} = \frac{a}{\sigma_\alpha \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{ax + b - \mu_\alpha}{\sigma_\alpha} \right)^2} \quad (15)$$

We get:

$$\begin{cases} a = \frac{\sigma_\alpha}{\sigma_\beta} \\ b = \mu_\alpha \left(1 - \frac{\mu_\alpha \sigma_\beta}{\sigma_\alpha \mu_\beta}\right) \end{cases} \quad (16)$$

Since the mass resolutions of the isotopes are assumed to be equal ³ $\left(1 - \frac{\mu_\alpha \sigma_\beta}{\sigma_\alpha \mu_\beta}\right) = 0$ and $\frac{\sigma_\alpha}{\sigma_\beta} = \frac{\mu_\alpha}{\mu_\beta}$. The linear transformation is then:

$$\mathcal{L}_{\alpha,\beta} T_\alpha(x) \equiv \frac{\mu_\alpha}{\mu_\beta} \cdot T_\alpha\left(\frac{\mu_\alpha}{\mu_\beta} x\right) \quad (17)$$

One of the things to notice is that this transform depends solely on the mean values of the distributions which correspond to the isotopes mass numbers.

3.3 Iterative Process

Once we have defined the linear transformation we can establish an iterative process to find one of the mass templates. We start by realizing that an experimental mass distribution, $D(x)$, where two isotopes are present, corresponds to the sum of two mass templates weighted by their relative fractions. .

$$D(x) = f_\alpha T_\alpha(x) + f_\beta T_\beta(x) \quad (18)$$

Making use of the transform:

$$\mathcal{L}_{\alpha,\beta} D = f_\alpha \mathcal{L}_{\alpha,\beta} T_\alpha + f_\beta \mathcal{L}_{\alpha,\beta} T_\beta = f_\alpha T_\beta + f_\beta \mathcal{L}_{\alpha,\beta} \mathcal{L}_{\alpha,\beta} T_\alpha \quad (19)$$

$$\Rightarrow T_\beta = \frac{1}{f_\alpha} \left(\mathcal{L}_{\alpha,\beta} D - f_\beta \mathcal{L}_{\alpha,\beta} \mathcal{L}_{\alpha,\beta} T_\alpha \right) \quad (20)$$

Replacing T_β in equation (19) with the expression on (20) and solving for T_α ,

$$T_\alpha = \frac{1}{f_\alpha} \left(D - \frac{f_\beta}{f_\alpha} \mathcal{L}_{\alpha,\beta} D \right) + \left(\frac{f_\beta}{f_\alpha} \right)^2 \mathcal{L}_{\alpha,\beta} \mathcal{L}_{\alpha,\beta} T_\alpha \quad (21)$$

This expression could be used to implement the iterative method but we can simplify it further by contracting the application of two linear transforms into one.

Applying 2 times the Linear Transformation

After applying the 1st transform, $\mathcal{L}_{\alpha,\beta}$, we end up with T_β which has mean value $\mu_\beta = \frac{1}{a} \mu_\alpha$. After applying the 2nd transformation we end up with a final function with mean value $\mu_f = \frac{1}{a} \mu_\beta$. We can relate the two mean values by subtracting one equation from the other, taking into account that $a = \frac{\mu_\alpha}{\mu_\beta}$, obtaining:

$$\mu_f = \mu_\beta + \frac{1}{a} (\mu_\beta - \mu_\alpha) = \frac{\mu_\beta^2}{\mu_\alpha} \quad (22)$$

Applying twice the transformation ($\mathcal{L}_{\alpha,\beta} \mathcal{L}_{\alpha,\beta}$) is equivalent to applying a single transformation,

$$\mathcal{L}_{\alpha,\beta} \mathcal{L}_{\alpha,\beta} \equiv \mathcal{L}_{\alpha,f} \quad (23)$$

where $\mathcal{L}_{\alpha,f}$ is the result of substituting μ_β for μ_f in equation (17). We are in condition of establishing an iterative process to get one of the templates, after defining a pair of guess fractions $\{f_\alpha, f_\beta\}$. The term of 0-th order is:

$$T_\alpha^{(0)} = \frac{1}{f_\alpha} \left(D - \frac{f_\beta}{f_\alpha} \mathcal{L}_{\alpha,\beta} D \right) \quad (24)$$

Terms of higher order can be calculated with:

$$T_\alpha^{(k+1)} = T_\alpha^{(0)} + \left(\frac{f_\beta}{f_\alpha} \right)^2 \mathcal{L}_{\alpha,f} T_\alpha^{(k)} \quad (25)$$

where the superscript represents iteration number/order ($k = 0, 1, 2, \dots$).

3.4 Mass Model

After finding the root template, the other(s) can be found by transforming the first and we can also construct a model from the calculated templates.

$$M(x; f_\alpha, f_\beta) = f_\alpha T_\alpha(x) + f_\beta T_\beta(x) \quad (26)$$

that can later be compared to the experimental data. We try different sets of guess fractions and for each compare the model with the experimental data via a χ^2 estimator. This estimator, in the case of histograms, is the sum of a quantity calculated for each bin according to,

$$\sum_{i=1}^n \frac{(N_D^i - N_M^i)^2}{|N_k^i|} \quad (27)$$

where N_D^i and N_M^i are the i -th bin content of the experimental data and the model, respectively and,

$$k = \begin{cases} D, & N_M = 0 \wedge N_D \neq 0 \\ M, & N_M \neq 0 \end{cases} \quad (28)$$

We expect that when the set of guess fractions corresponds to the real ones the model obtained will optimize the match with data, corresponding to a minimum of our estimator.

4 DDM applied to continuous functions

In order to better understand the DDM behaviour, we start applying it to a continuous mass distribution made of two gaussian mass functions, weighted respectively by f_1 and f_2 with $f_1 + f_2 = 1$. We set the gaussian means corresponding to the isotope masses to $\mu_\alpha = 1$ and $\mu_\beta = 2$ and the standard deviations to $\sigma_\alpha = 0.2$ and $\sigma_\beta = 0.4$. The later fulfills the equal mass resolution condition $\frac{\sigma_\alpha}{\mu_\alpha} = \frac{\sigma_\beta}{\mu_\beta}$. The "real" fractions (the weights of the two gaussians) were set to ($f_1 = 0.3, f_2 = 0.7$). Figure 4 shows the total mass distribution and the individual mass templates.

³ $\frac{\sigma_\alpha}{\mu_\alpha} = \frac{\sigma_\beta}{\mu_\beta}$

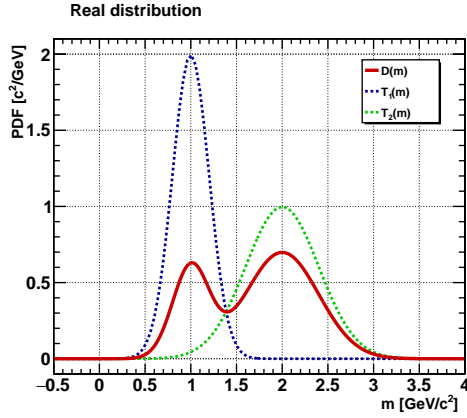


Figure 4: Real distribution with fractions ($f_1 = 0.3, f_2 = 0.7$).

As stated before (section 3.4) the derivation of the appropriate isotopes fractions on mass distribution requires searching for a minimum in the parameters space (f_1, f_2) of the χ^2 estimator. The mass templates that compose the mass model are, according to equation 25, function of (f_1, f_2). Figure 5 shows the mass templates derived after 2 iterations for wrong guess fractions ($f_1 = 0.2, f_2 = 0.8$).

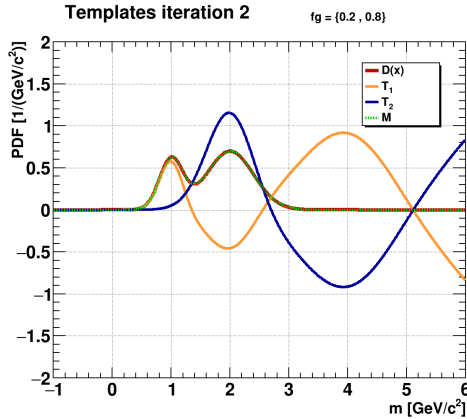


Figure 5: Data modeling function (D), isotope mass templates weighted by the incorrect guess fractions ($f_1 = 0.2, f_2 = 0.8$) derived after 3 iterations (T1, T2) and mass model (M).

One can observe that exist mass oscillations for masses higher than $\mu + 3\sigma$ on both templates. However, for the correct guess fractions ($f_1 = 0.3, f_2 = 0.7$), mass oscillations vanish as can be seen on figure 6.

Further analysis, (see Appendix B), leads to 3 conclusions concerning the amplitude of the mass tails oscillations:

- almost vanishes when the guess fraction matches the real fraction ($f_{1,g} = f_{1,r}$)
- increases with iterations if the guess fraction ($f_1 < 0.5$)
- decreases with iterations if the guess fraction ($f_1 > 0.5$)

This was achieved by testing 3 different sets of guess fractions (with $f_{1,g} = \{0.2, 0.3, 0.7\}$) for two distinct cases

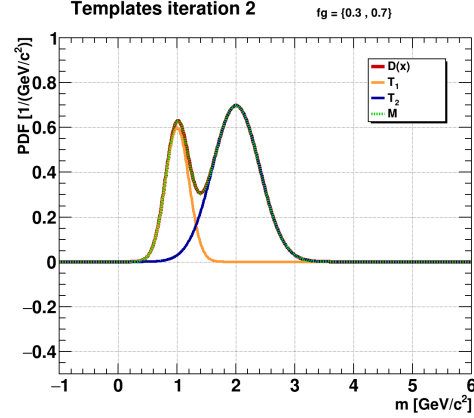


Figure 6: Model, weighted mass templates and true distribution after 3 iterations for the correct set of guess fractions ($f_1 = 0.3, f_2 = 0.7$).

concerning the real fraction of the first isotope, $f_{1,r} = \{0.3, 0.6\}$. The fact that the amplitude of the tails almost disappears when the fraction corresponds to the real one, will prove essential to determine the correct fraction.

We also notice that in the region of small mass values $[0, 6]$ GeV/ c^2 the models converge after few iterations (see Zoomed plots on Appendix B).

When we scan the parameters space, building a model for each set of guess fractions and calculating the χ^2 after a number of iterations necessary for convergence in the small mass region, $m = [0, 6]$ GeV/ c^2 , we are unable to identify a minimum of the estimator corresponding to the correct fraction. This is shown in figure 7, where we plot the χ^2 for a simulation in which the real fractions were set to be $f_1 = 0.5$ and $f_2 = 0.5$ after 4 iterations.

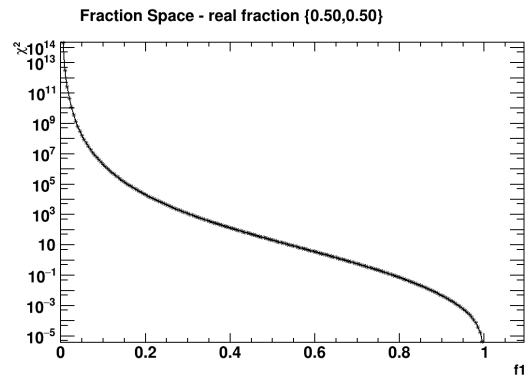


Figure 7: χ^2 as a function of the guess fraction of the first isotope, f_1 , after 4 iterations.

We would expect to see a minimum of the χ^2 when f_1 would match the real fraction (in this case 0.5), but we see that the estimator value keeps decreasing as the guess fraction increases.

DDM features

Here we summarize some of the features we have to take into account of the DDM when applied in its raw format:

1. The template bins acquire negative values, which have no physical meaning
2. We cannot identify a minimum of our estimator- χ^2 in the fraction parameters space
3. The models obtained have "tails" whose mass values increase (exponentially) with the iteration. The problem of sectioning the histogram arises. To what extent does it make sense that our model exists?

5 Absolutifying Templates

In order to solve the first and second features mentioned in the previous subsection we proceed to "absolutify" the templates (ABS option), which means, that we loop through all the bins of our histograms (or function values) and if the bin content is negative we take its absolute value. With this procedure we can identify a local minimum of the $\chi^2(f_1)$ that corresponds to the real fractions (see figure 8).

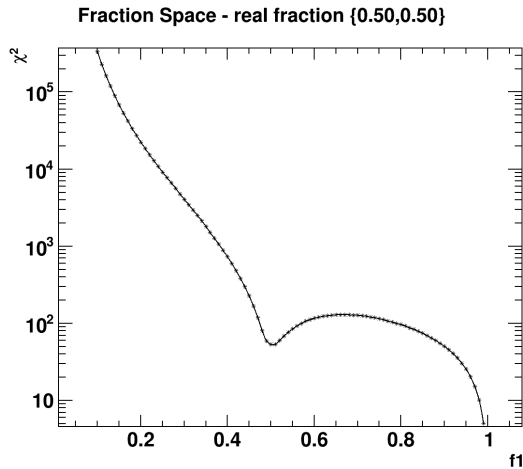


Figure 8: χ^2 as a function of the guess fraction of the first isotope, f_1 , with ABS option.

The feature that enables us to identify this local minimum is the tails the models acquire. This conclusion comes from calculating the value of the estimator for two different mass regions (figure 9) of our templates:

1. Region 1 - where the experimental mass distribution has positive values
2. Region 2 - region of the oscillating mass tails assuming negative and positive values

Identifying this feature is important because it enables us to identify the real fraction, $f_{1,real}$, even when it has values greater than 0.9, a situation in which even with the ABS option it is not possible to clearly identify the minimum of the estimator.

Higher Fractions

When one of the real fractions of our distributions is close to 1 (e.g. $f_{1,real} = 0.95$), values of the χ^2 in the two regions are both small and of the same order (notice vertical scale on figure 9), which means that when we add them up we lose the information about the minimum. We can calculate our total χ^2 giving a higher weight to tails region (2)⁴ to enable identification of the minimum.

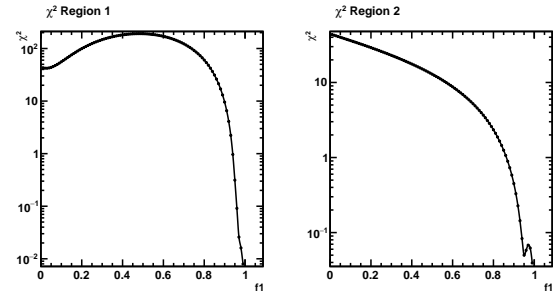


Figure 9: χ^2 calculated for each region after 4 iterations. The real fraction of the first isotope was set to be 0.95.

6 Results and Conclusions

Results

The final mass templates will be obtained by setting to zero the non-physical mass oscillating tails⁵. Figure 10 shows the kind of correspondence between the templates extracted with the DDM and the simulated data that can be obtained.

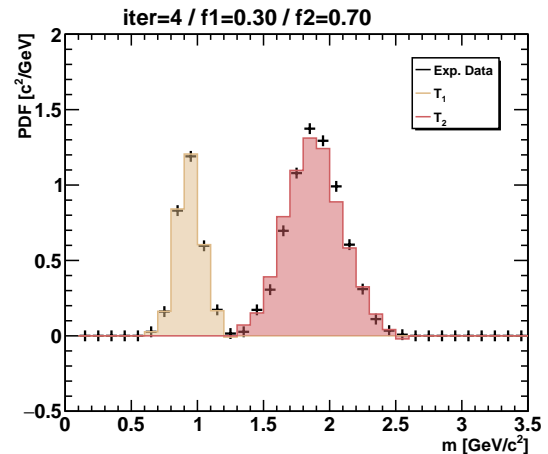


Figure 10: Mass templates for the proton and deuteron obtained with the DDM against the simulated data.

Conclusions

In this project, it was made a Monte Carlo simulation replicating the detection of events in the AMS detector, in order

⁴ $\chi^2 = \chi_1^2 + w \cdot \chi_2^2$

⁵Since at this point templates are not normalized, normalization is also imposed

to obtain mass distributions of isotopes with a given charge number, Z .

We have implemented a data driven method to extract mass templates for each isotope present, which works in two steps: first, finding the isotopes' fractions; second, get the mass templates and construct a model of mass distribution. In order to find the isotopes fraction we test various possible fractions (guess fractions) and construct absolute templates. The right fraction corresponds to a local minimum in the χ^2 fraction space. The final mass templates are obtained by setting to zero the tail oscillations allowing us to obtain a good agreement of the model with the simulated mass distribution.

Acknowledgements

Thanks to all LIP team members involved in providing this experience to us interns. Thanks also to Miguel Orcinha and Rafael Parente for their helpful insights and discussions on some of the ideas here presented, and for good companionship. Huge gratitude to Fernando Barão for the same reasons but also for introducing me to new tools and transmitting to me values and good practices on collaboration work.

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A C++ Classes

To implement the data-driven method several C++ classes were developed. The most important are:

- **uIsotopesMC** - simulate the detection of events in the TOF and RICH detectors of the AMS from isotopes of a given charge, Z . Stores the information in TTree object
- **uIsotopesAnalysis** - developed to store relevant histograms constructed from data stored in a uIsotopesMC object

- **uTransform** - has methods to perform linear transformations on ROOT objects TF1 and TH1D (functions and histograms).
- **uIsotopesDDM** - receives an experimental/simulated mass distribution; has methods to perform each iteration of DDM and stores the mass templates and models in a map object.

B Testing with Functions

In this appendix section we show schematics of the templates and model evolution with iteration for two different sets of real fractions and various guessing fractions. Three iterations are shown for each guessing fraction. It is shown the graph as well as a zoomed in version. Notice the change in scale in both axis.

- $f_{real} = \{0.3, 0.7\}$

– $f_g = \{0.2, 0.8\}$

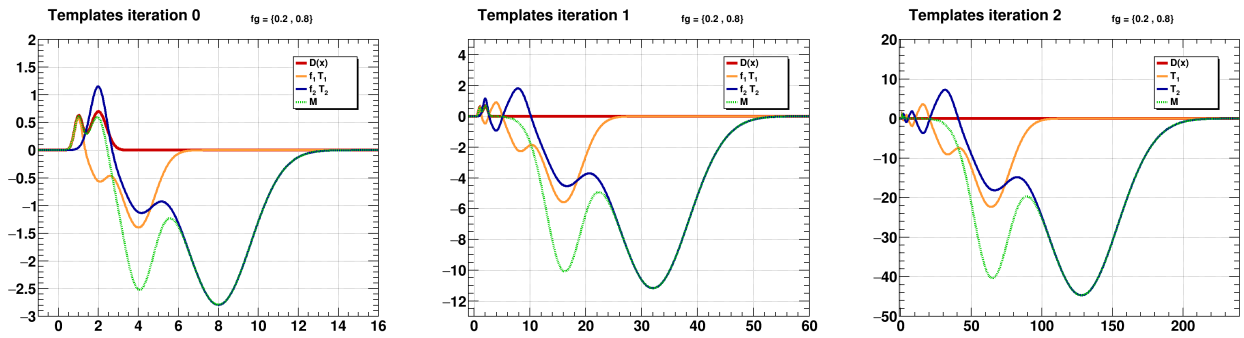
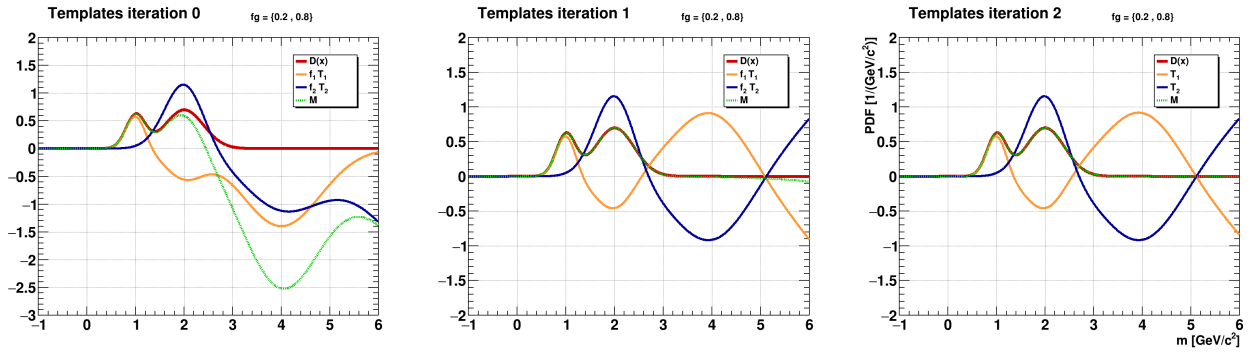
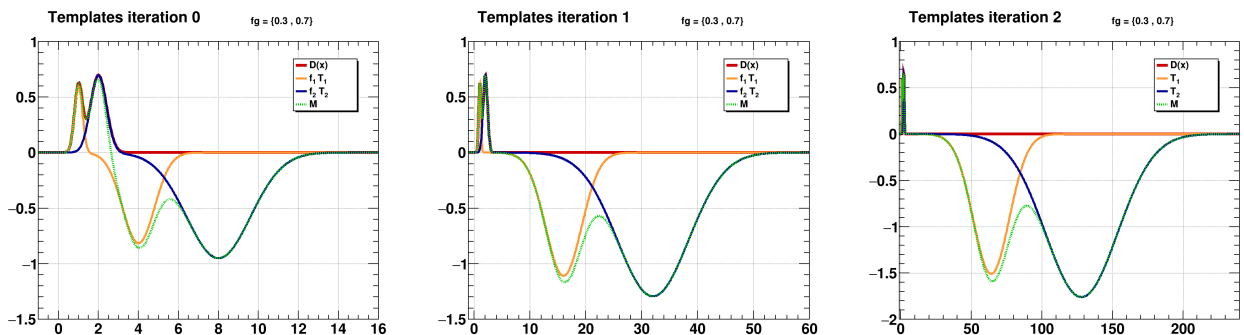


Figure 11: Three simple graphs

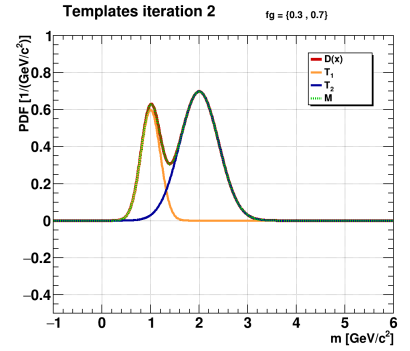
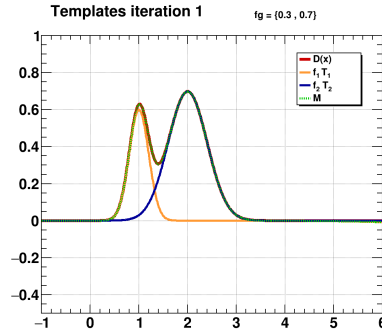
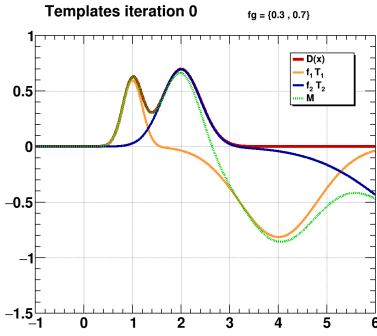
Zoomed



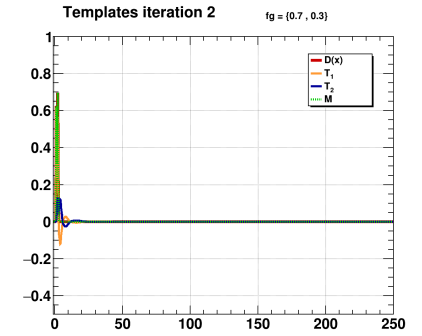
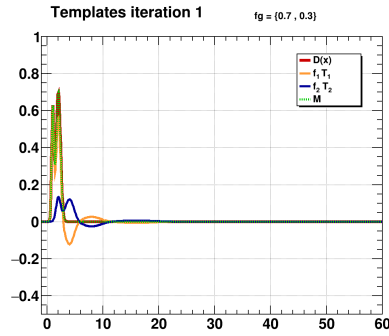
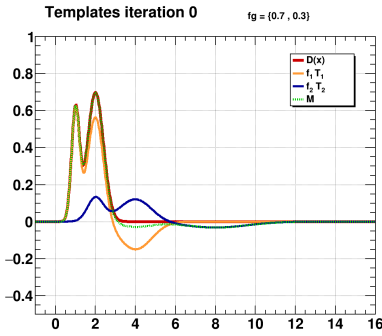
– $f_g = \{0.3, 0.7\}$



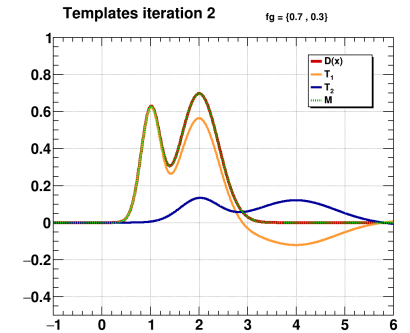
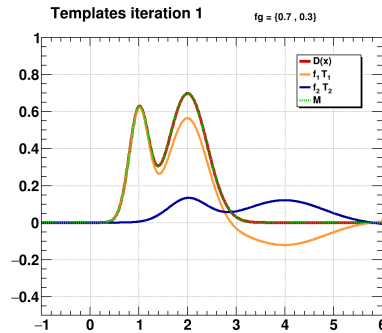
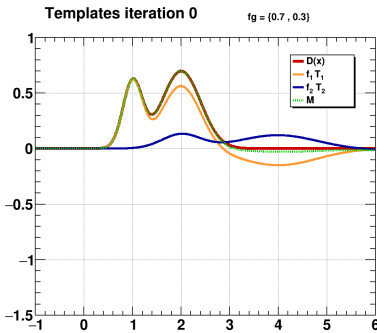
Zoomed



– $f_g = \{0.7, 0.3\}$

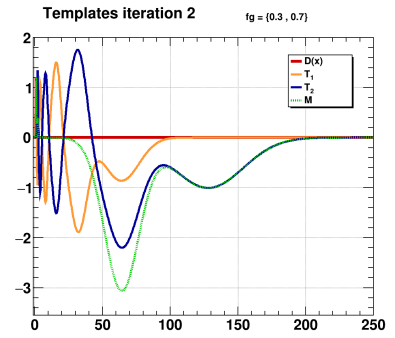
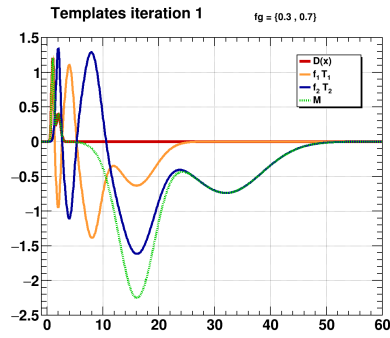
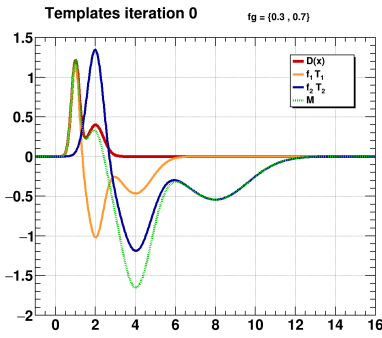


Zoomed

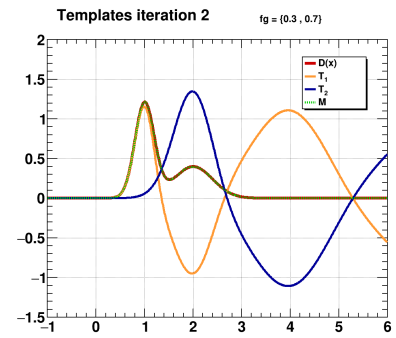
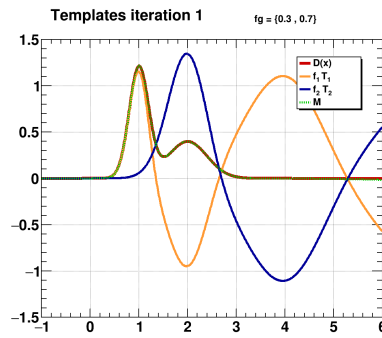
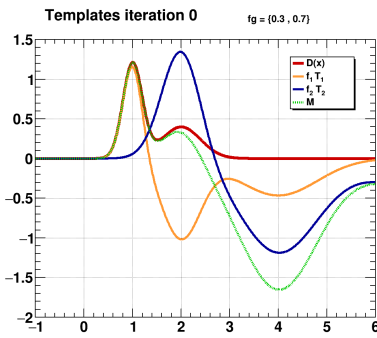


• $f_{real} = \{0.6, 0.4\}$

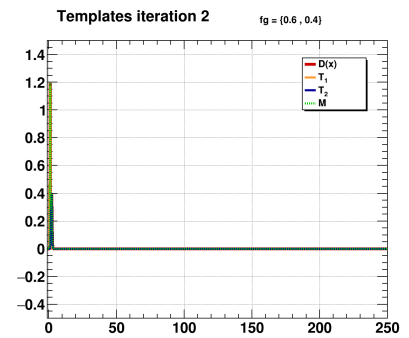
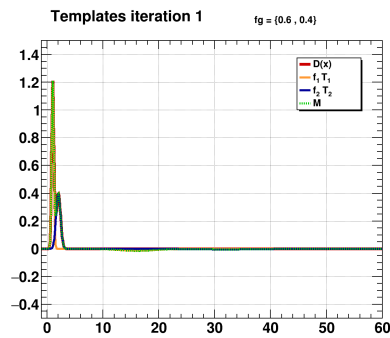
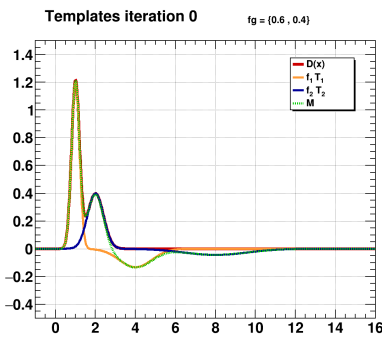
– $f_g = \{0.3, 0.7\}$



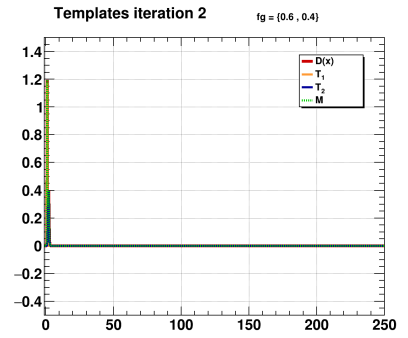
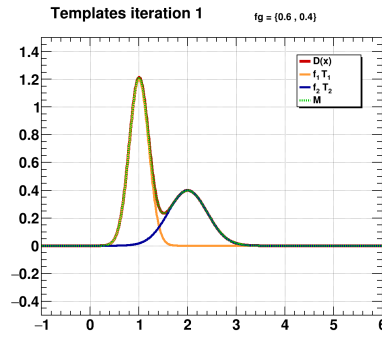
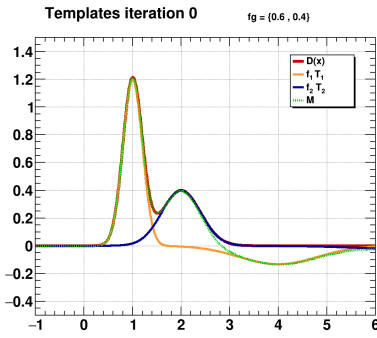
Zoomed



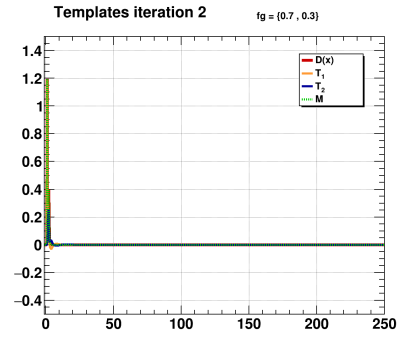
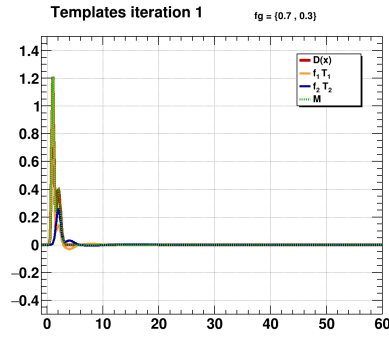
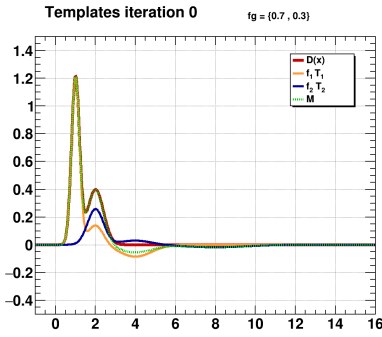
– $f_g = \{0.6, 0.4\}$



Zoomed



– $f_g = \{0.7, 0.3\}$



Zoomed

