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Structure in the EAS size spectrum: analysis of the CASA-MIA results

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Abstract

We present an analysis of the very recent data from the CASA-MIA extensive air shower array (M.A.K. Glasmacher, Ph.D. Thesis, University of Michigan (1998)) in terms of a search for the structure in the shower size spectrum claimed by us from an analysis of "the world's data". Our earlier claim is found to be supported to the extent that there is strong evidence for the existence of structure in the spectrum which cannot obviously be explained by the conventional Galactic Modulation Model. There is modest evidence for the structure being of "our" form and strong support for "our" mass composition when "corrected" to the interaction model advocated by us. None of the results are inconsistent with there having been a recent, nearby, single supernova. © 1998 Elsevier Science B.V.

1. Introduction

Although it is 60 years since extensive air showers were discovered and 40 years since the Moscow group [8] showed that the important "shower size spectrum" at ground level had a pronounced increase of slope (at a size corresponding to a primary energy of about 3×10^6 GeV) – this feature being referred to as the "knee" – further advances have been slow in coming. We claim to have made such an advance, however, by way of a demonstration of structure in the size spectrum in the region of the knee. A series of papers [2–5] has given detailed evidence for the structure and a possible interpretation. Supporting evidence has come from studies of the changing phases and amplitudes of the anisotropy [6] and from a detailed analysis [7] of the mass composition of the primary particles by way of measurements on the depth of shower maximum and on the muon-to-electron ratio. In fact, the analysis of the mass composition enabled previous inconsistencies [7] to be resolved and a preferred interaction model to be put forward.

Crucial support for our model – at least in our view – has come from the fact that the directly measured intensities at lower energies, extrapolated by only one or two decades, meet the spectra postulated by us to explain the claimed structure in the size spectra [6]. It is self-evident that there must be structure in the size spectra *at some level* insofar as the directly measured spectra for the various mass-groups: CNO, Ne–S (denoted H) and Fe would, if extrapolated, "poke through" the total spectrum if not terminated. It is the sharpness of the termination that will give the structure; if the terminations *are* sharp then the structure will be considerable, if not then the structure will be less marked, although still present to some degree.

In our previous work we claimed that there was in-

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deed significant structure and this was interpreted in terms of the presence of a single source, which gave a rather large fraction of the particles over a limited size region. Because the structure coincided with that expected for a single young supernova remnant accelerating nuclei from the hot interstellar medium (see [2] for details of the SNR model), we put forward this young SNR as the basis of the model. Of course, there will be other possible mechanisms but we still regard the structure in the size spectra as demanding a unique event of some form.

The arguments are, essentially, to do not with the Astrophysics but with the spectral shape and its structure; the question to be addressed is whether or not it is as claimed by us.

It must be stated at once that there has been little sympathy, so far, for our view (e.g., [9]) although some workers (e.g., [10,11]) are more open minded than others.

The recently published results [1] of the CASA-MIA array, with their unrivalled statistical accuracy, offer the possibility of checking on our claim; this is the purpose of the present work.

Before starting, it must be made clear that there are many problems in comparing size spectra determined by different workers with different arrays; indeed, comparison of size spectra by the same array at different zenith angles is not straightforward (e.g., [5]). For a start, the quantities measured which are used to give the "size-spectrum" are not always the same. For example, some (and CASA-MIA is in this category) have lead above their detectors in order to "materialise photons" and thereby increase the number of detected particles. The manner of combination of the detector-counts differs from experiment to experiment, too. The result is that there will be systematic differences in the results of one experiment with respect to another. Nevertheless, we argue that by "standardising" the size spectra at the knee and putting increased emphasis on the *pattern* of the size spectrum near the knee – rather than the absolute value of our claimed excursions – results from different arrays *can* be compared.

2. Analysis of the CASA-MIA results

2.1. The size spectrum

2.1.1. Identification of the knee position

It is well known that different EAS arrays, even those operated at the same atmospheric depth, give knee positions at different sizes [2–5]. Often, the knee is defined as the crossing point of two straight lines drawn below and above the apparent knee but this can be imprecise in view of the frequently measured spectral curvature below the knee, complications near the knee and above, and often a limited range of sizes above the knee. (Nevertheless, we will make some analyses using this crossing point, the size for which we term N_e^{CP}).

Another technique, that we used earlier [2–5], is more appropriate when the statistical accuracy is reasonably good. This is the use of the sharpness index, S ($S = -\partial^2 \log I(N_e)/(\partial \log N_e)^2$), the "knee" is then defined as the first (significant) peak in S versus $\log N_e$. It is reassuring [5] that, when the knee point is plotted against atmospheric depth for the various arrays, the dispersion about expectation is somewhat better than that for the crossing points. It should be added that in [5] we presented evidence which went some way towards explaining the reason for the spread in knee-positions (however defined).

Other techniques can also be used. One – which can be used when the data are of high statistical precision – is to pick the first "peak" in the size spectrum (plotted as $N_e^3 I$ vs N_e), and identify it with the "knee". Another method is to draw a polynomial through the intensities and identify the knee with the position of maximum curvature (denoted N_e^{MC}). This technique is of value when the data are available with the same array at different zenith angles. The object then is to compare the spectra from different zenith angles by standardising to the same knee position. If a consistent pattern is seen from one zenith angle to another then the case for *structure* in the size spectrum is considerably strengthened, irrespective of what the structure is due to (O, H, Fe ...) and irrespective as to whether or not the knee is, in fact, due to oxygen.

In what follows we use the peak identification technique for the highly precise near-vertical data and the maximum curvature data in the inclined directions (with the crossing point method as a check).

2.1.2. The experimental array and results for near-vertical showers

This is not the place to describe the CASA-MIA array, and its results, in detail – these are given in [1]. Suffice it to say that some 5.4×10^7 showers were recorded and "electron" size spectra have been given for 7 zenith angle bins, covering the range of zenith angles (θ) given by $\cos\theta$: 1.0–0.97; 0.97–0.94 ..., 0.82–0.79. The corresponding atmospheric depths range from ~ 860 g cm^{-2} to 1087 g cm^{-2}.

We start by considering measurements in the most nearly vertical zenith angle range ($\theta < 14°$) and with the smallest bin size ($\Delta \log N_e = 0.05$). The differential size spectrum from [1] is given in Figs. 1a and 1b.

It is evident from these figures that there *is* structure in the size spectrum over and above the (statistical) errors indicated. It is difficult to think of systematic errors which would give such large features when there are only small ones at smaller sizes.

It is also evident that the knee is not very sharp. Specifically, the magnitude of the maximum sharpness is only 1.1; whereas in [5] we found a median sharpness, from 17 sets of data (from 11 arrays), of 2.1; only one experiment (Tibet) had a lower value than this: $S_{\max} = 0.8$. It is not at all clear why these two experiments give such small maximum sharpnesses. Nevertheless, both values (0.8 and 1.1) are well above the value of 0.3 estimated by us [5] for the commonly adopted model of "Galactic Modulation". In other words, there *is* a significant "knee".

Fig. 1a can be considered still further. The dashed lines for sizes above $\log N_e = 6.6$ are our estimates of the likely range in this region, a region for which the CASA-MIA results become increasingly insecure. The upper line, marked (1), is the average (in terms of slope) for the world survey in the region above the knee starting at $\log N_e^{\text{knee}} + 1.0$. Actually, only Tien–Shan (Hadron), Akeno and MSU arrays have measurements in this region (see [5]) and their average slope is 2.76 ± 0.04. The lower line (2) is where we would expect the CASA-MIA spectrum to be in the region above $\log N_e = 6.6$ if it were as different from the world-average as is the spectrum *before* the knee (the latter has slope 2.70 compared with the world average there of 2.55). The logic here is the fact that in [5] we demonstrated that the dispersion, experiment to experiment, of the difference in slopes, above and below the knee, was smaller than that in either; a possible reason for this effect was put forward [5].

Examination of the overall spectral shape (in Fig. 1a) now shows that there is no spectrum with gradually increasing slope – such as would be expected for Galactic Modulation [5] – that will fit the data. Thus, there is structure in the size spectrum of some sort.

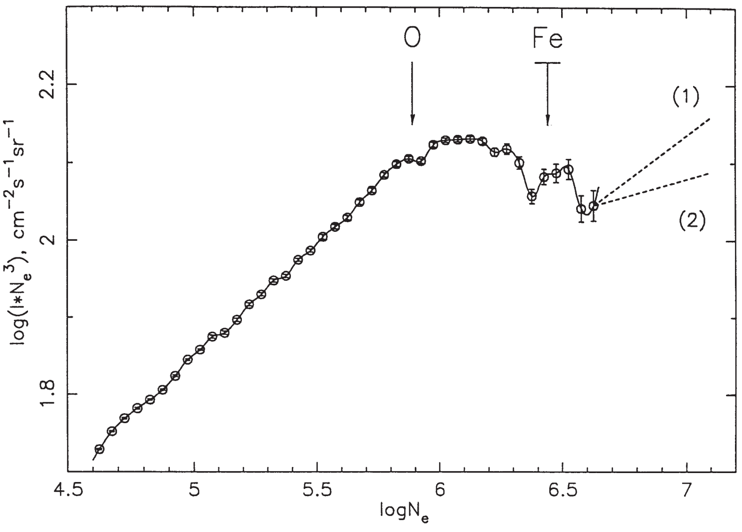
Despite our inability to fit a smooth curve to Fig. 1a, we can consider the extent to which a smooth curve would fit the measurements in the knee region alone. Fig. 1b shows such a curve fitted to the range $\log N_e =$ 5.65–6.65. Accepting the errors at their face value the value of χ^2 is 56 for 20 points and the probability of a fit is vanishingly small ($\sim 2 \times 10^{-5}$), so the null hypothesis which assumes the smooth steepening of the CASA-MIA EAS size spectrum with no fine structure in it is to be rejected.

We can go further and consider the extent to which the structure in Fig. 1b resembles that claimed by us in our World Survey [5]. This is best done in terms of the deviations from the running mean because of the different sensitivities of the different arrays to the magnitude (but not the shape) of the structure. An analysis of this aspect is given later.

Returning to Figs. 1a, 1b we have indicated where *we* estimate the positions of the "oxygen" and "iron" peaks to be, based on inspection of the spectrum itself, identifying the first peak with oxygen and placing the iron expectation position at a size higher by $\Delta \log N_e = 0.55 \pm 0.05$. The value 0.55 ± 0.05 needs some explanation. In [5] we pointed out that there was evidence for its dependence on the slope of the size spectrum before the knee, for technical reasons, and it should vary slightly with atmospheric depth and zenith angle at the same location. Thus, a significant range of uncertainty must be allowed and the "preferred" value from inspection of [5] is 0.55. The size spectra are clearly not inconsistent with our predicted "second peak" – if as appears likely, our estimate of the knee position is correct.

To reiterate the reasons for identifying the "peaks" with O and Fe (as distinct from other elements), their positions are, roughly, where the extrapolated directly measured spectra would appear, and they are also just where the SNR model would have them be (see [5]).

a) CASA–MIA EAS SIZE SPECTRUM IN THE VERTICAL DIRECTION



b) Expanded scale, 3 order polynomial fit, RMS of the fit = 1.67

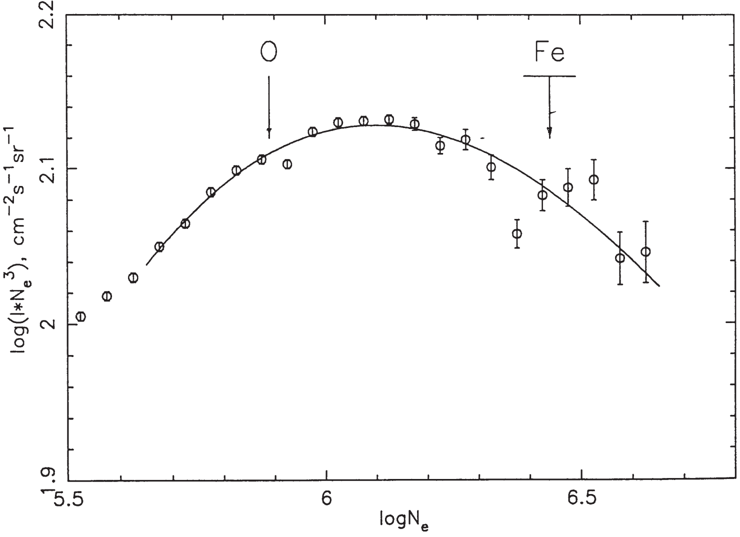


Fig. 1. (a) CASA-MIA size spectrum in the near vertical direction (zenith angles $< 14^\circ$) [1]. The line is drawn between the points. (1) and (2) represent likely limits on the spectrum beyond $\log N_e = 6.6$. “O” and “Fe” represent our estimate of the positions of the oxygen and iron “peaks”. (b) An enlarged version of (a) for a restricted region of size. The line is a 3-order polynomial fit symbolising a possible “Galactic Modulation” prediction. It is apparent that it does not fit the points. (c) Excesses from the running mean for two bins over which the points are averaged (as indicated). The positions of the O- and Fe-peaks are from (a). The near horizontal chain line, which starts at $\log(N_e/N_e^{\rm knee}) \simeq -0.2$, is derived from the 3rd order polynomial fit in (b).

In passing, it can be remarked that the case for a preponderance of oxygen in the CNO group was made in [3].

In our previous work [4] we quantified the excesses by using the running mean, viz. taking each point in turn and comparing its intensity with the average over ±0.25 in $\log N_e$. Here, with the higher precision we can take smaller bins, particularly for Fig. 1a, because

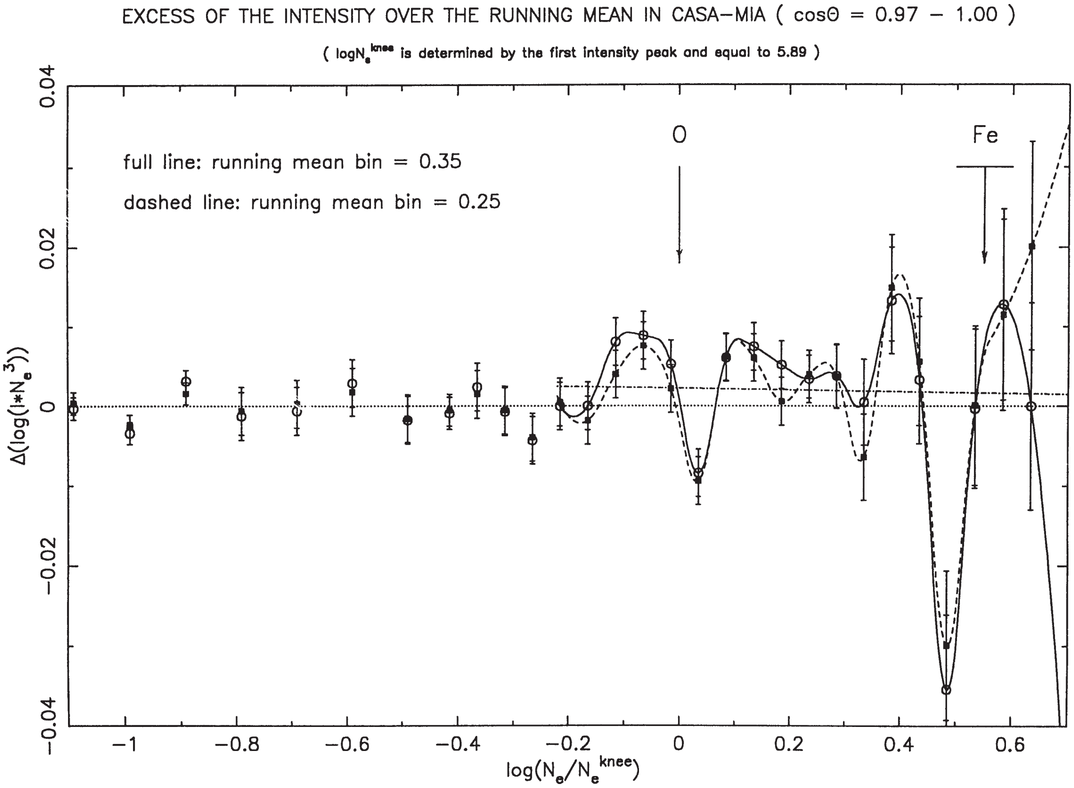


Fig. 1 — continued.

the points are spaced by 0.05. In order to examine the sensitivity to width of averaging we have studied two situations: ±0.125 and ±0.175. The results are given in Fig. 1c; there is seen to be excellent agreement between both variants. That proves the insensitivity of the result to the choice of the bin width for the running mean method.

It is necessary to comment on the validity of using the running-mean technique. Whilst it is true that adjacent points are correlated, this correlation is very small, the reason being that the mean comes from five points (or 7). The value of the running mean is that it enables a search for structure in the presence of a base level which may well be variable from one experiment to another (and one zenith angle to another in the *same* experiment). It is reassuring that the χ^2-value for the fit of the smooth curve in Fig. 1b is similar to that for the corresponding curve in Fig. 1c: 48, for 18 points. This justifies the use of the running mean technique.

Turning to the positions of the peaks in the right-hand half of Fig. 1c, the O-peak is as defined by us (see later for details), and, as before, the Fe-peak is indicated at an abscissa higher by 0.55 ± 0.05. The evidence for the Fe-peak is not strong, but, bearing in mind the fact that the negative excursion adjacent to the peak is also relevant, the case is reasonable for there being a peak at least not far from our expectation. Some slight evidence exists for peaks between O and Fe; these were identified by us in the "world survey" as perhaps being due to "heavy nuclei" (Ne–S), insofar as this is where this group of nuclei should appear. However, we are not claiming their detection.

The strongest case for the existence of structure in the CASA-MIA experiment comes from the basic differential size spectrum (Fig. 1a). This spectrum has, of course, independent points. As remarked earlier, there is no Galactic Modulation spectrum that can be drawn which leads smoothly from the region below the knee ($\log N_e < 5.7$) to that well above the knee ($\log N_e > 6.7$).

If attention is confined to the range $\log N_e$: 5.7 to 6.7 and a smooth line is drawn in this region alone,

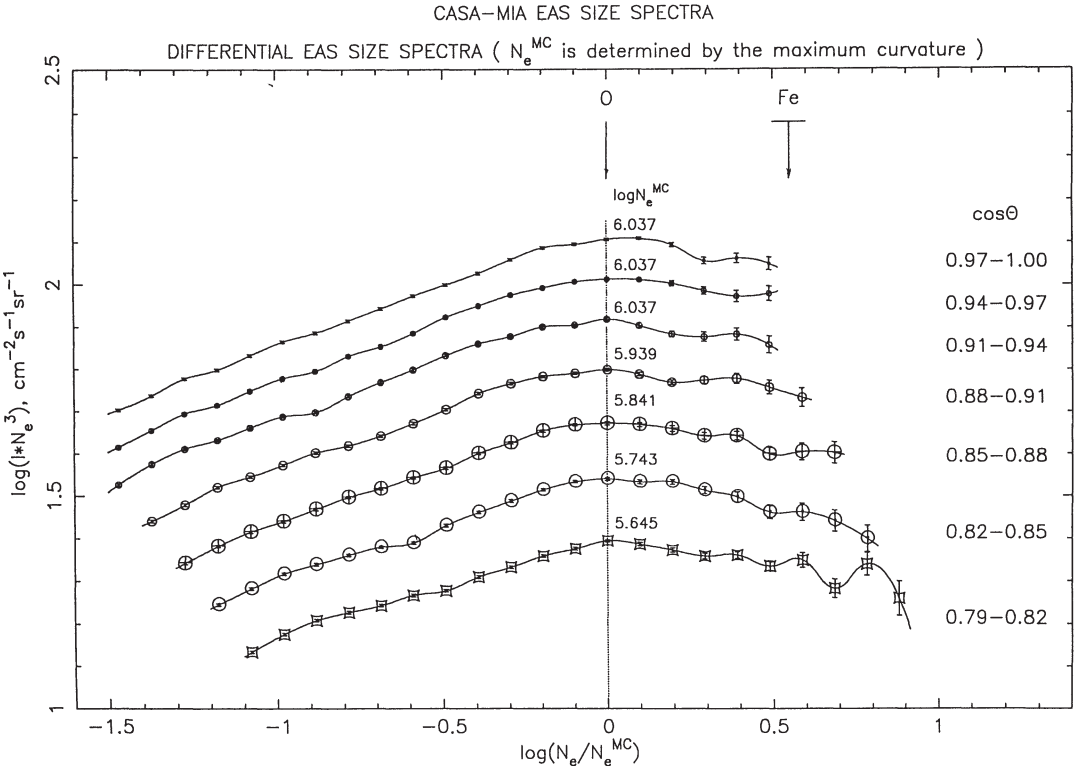


Fig. 2. (a) CASA-MIA size spectra displaced by us so as to give the same size for the value of N_e (denoted N_e^{MC}) for maximum curvature. The actual values of $\log N_e^{\mathrm{MC}}$ are indicated. The O-peak is at zero on the abscissa (by definition) and the Fe-peak is at 0.55 ± 0.05 (see text). (b) Excesses from the running mean (5-point) for all the data given in (a). The symbols relate to the various zenith angle ranges given in (a). (c) The weighted mean excesses derived from (b).

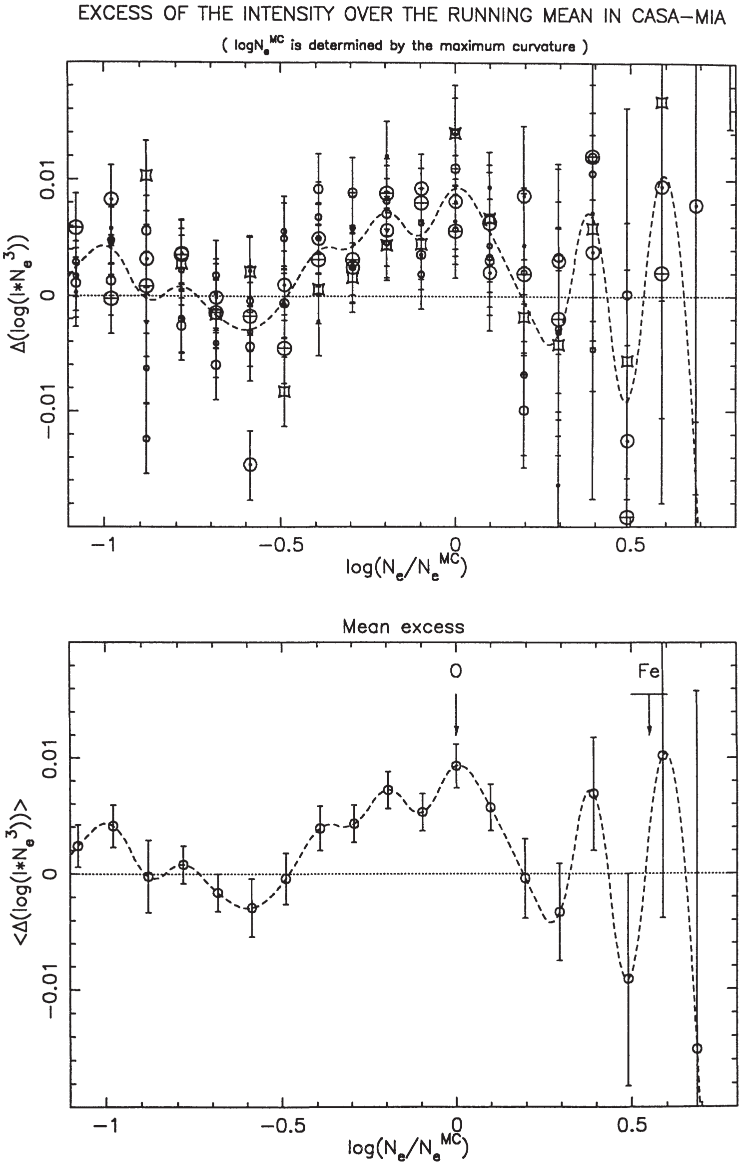
the fit is impossibly bad.

2.1.3. Development fluctuations

A point to be considered is the extent to which any "structure" can be real. Apart from instrumental effects, development fluctuations during the shower's propagation through the atmosphere set a limit. These have been estimated by many authors, very recently by the Karlsruhe group [12], with similar results. For primaries of unique energy, and for size measurements in the near-vertical direction near sea level, the results at 10^{15} eV are [12] FWHM: $\Delta(\log N_e) = 0.53 \pm 0.07$ for protons and 0.20 ± 0.02 for iron nuclei, these values being derived from plots of the frequency distribution of electron sizes for showers produced by primaries of the same energy. The quoted errors represent the range occupied by the different interaction models employed (DPM, QGSJET, SYBILL, etc.). It is not surprising that there is not much spread in the values for the different models insofar as most of the spread comes from purely statistical factors related to the longitudinal development.

The FWHM for showers at a particular depth falls with increasing primary energy, largely because the position of shower maximum becomes closer to the observation level. By the same token, the FWHM increases with increasing atmospheric depth for the same primary energy. This dependence has also been studied by various authors. Here, we need the results for a smaller atmospheric depth: CASA-MIA at 860 g cm^{-2}, compared with 1020 g cm^{-2} for the calculations in [12]. The result from [13], is that for CASA-MIA and small zenith angles, the widths will be smaller, ≈ 0.35 (P), 0.15 (Fe).

In so far as we are dealing with heavy nuclei, a bin size of $\Delta \log N_e = 0.1$ is justifiable if the statistics


EXCESS OF THE INTENSITY OVER THE RUNNING MEAN IN CASA–MIA

($\log N_e^{MC}$ is determined by the maximum curvature)

Mean excess

Fig. 2 — continued.

allow it; in the case of CASA-MIA they do.

2.1.4. Analysis of the size spectra at various zenith angles

The availability of precise data at a variety of zenith angles and with the *same* array affords a good opportunity to search further for the claimed structure. If the claimed structure shows up at all zenith angles then this can be regarded as quite strong confirmation of the pattern found in the vertical direction.

At the risk of overinterpreting the data, we endeavour to make an analysis of the inclined data.

In [1] the size spectra were binned more coarsely for the inclined directions than for the vertical direction, viz in $\log N_e$ bins of 0.1 (cf. 0.05 in Figs. 1a, b). Thus, another technique was used, rather than the identification of the small peak due to oxygen, specifically the maximum curvature method (see Section 2.1.2).

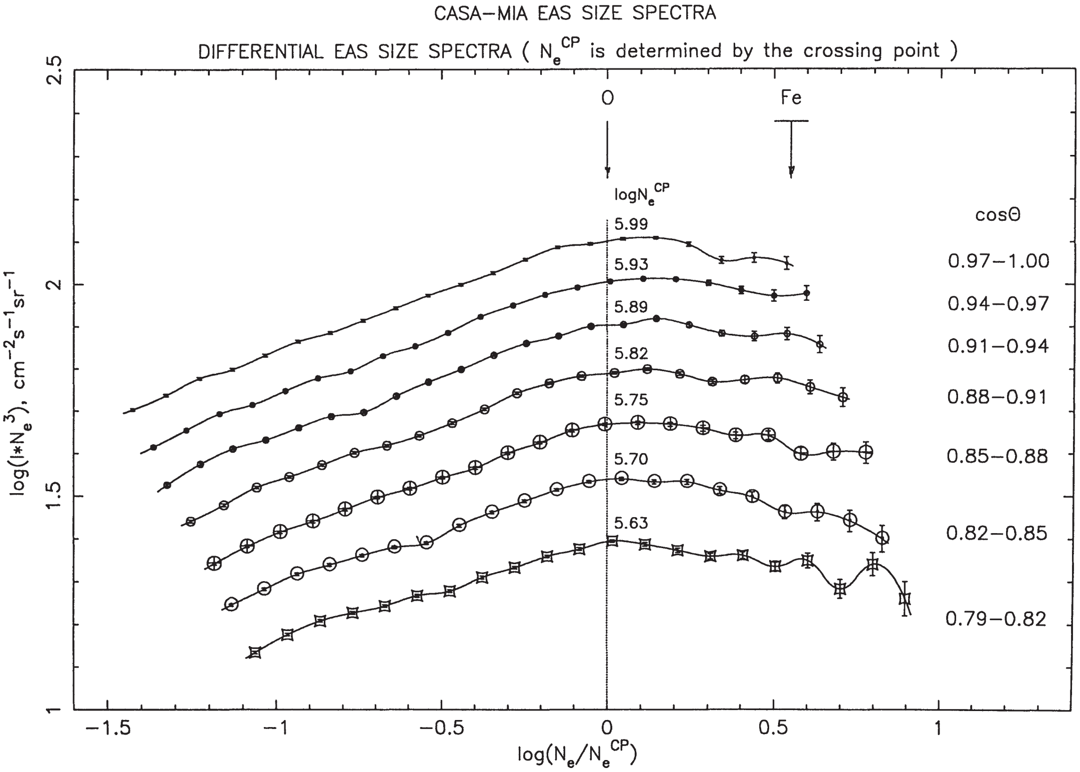


Fig. 3. (a) As for Fig. 2a but using the crossing point method (i.e., the intersection of two straight lines) to give the knee. (b), (c) As for Figs. 2b, c but for the crossing point method.

The results are shown in Fig. 2a. The corresponding "excess of the intensity over the running mean" is shown in Figs. 2b, c. The significance of plots will be considered later.

What is necessary to consider here is the distinction between the "O-peak" in Figs. 1a and b and the knee identified for the same, near-vertical showers, using the maximum curvature method in Fig. 2a. The difference ($\Delta \log N_e \simeq 0.15$) arises from the fact that in Fig. 1a, as already remarked, the bin width in $\log N_e$ is 0.05 to be compared with 0.10 with Fig. 2, thus, some fine structure was detectable in Fig. 1 that did not appear in Fig. 2.

Two reasons make us think that Fig. 2 is *probably* nearer the truth.

(i) All the patterns in Fig. 2a are of similar shape with this identification.

(ii) With the higher value of $\log N_e$ for the knee in Fig. 2a ($\log N_e^{CM} = 6.037$), the integral flux above the knee is nearer to expectation [5].

Although the displacement in knee position for the near-vertical showers may seem large (0.15), in fact, as can be seen in Fig. 1a, a displacement of the O-peak to the right by this amount still allows the Fe-peak to be just in the acceptable region.

As mentioned earlier, we have also used the "crossing point" method; the results in this case are shown in Fig. 3 (the crossing points were determined disregarding points within ±0.25 in $\log N_e$ of the knee). It is apparent that the differences in the "mean excess" values are not large, viz., the claim for structures is robust with respect to the method of determining the knee.

Inspection of Figs. 2c and 3c shows that if the datum were zero for the datum level, only the oxygen peak would be significant. However, there are small excesses (bracketed by deficits) at the Δ-values where the iron peak *should* be. Having said that, the only safe evidence for the iron peak is that given in Fig. 1.

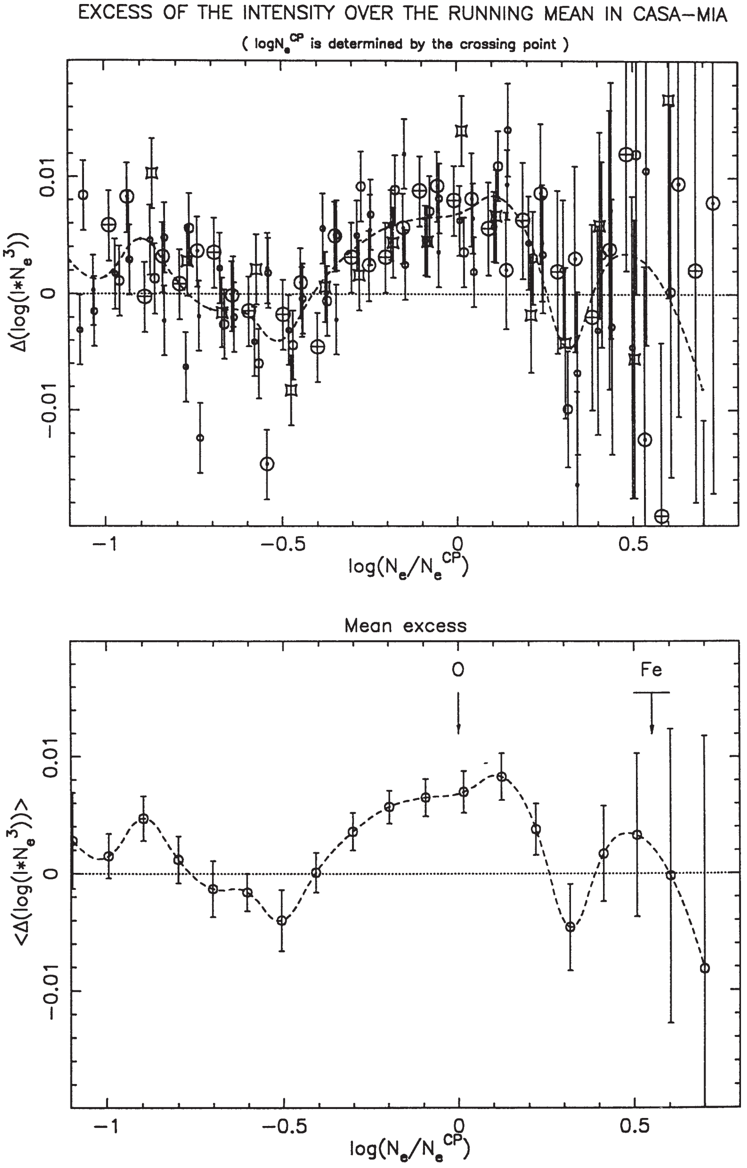


Fig. 3 — continued.

2.2. Quantitative analysis of the significance of the pattern

We return now to the important question of the extent to which the different sets of data in Fig. 2a are consistent one with another. An analysis has been made of the difference between each point and the corresponding derived mean for that bin. The difference is divided by the standard deviation, σ_0, determined from the error on the point and the error on the mean. The distribution of this quantity is given in Fig. 4. Comparison is made with expectation (which is approximately a Gaussian with standard deviation $0.73\sigma_0$ rather than σ_0 itself, because each point affects the running mean value). The result is seen to be rather good; only a very few points are in an anomalous tail to the distribution and these do not have a significant effect on the means in Fig. 2. We regard

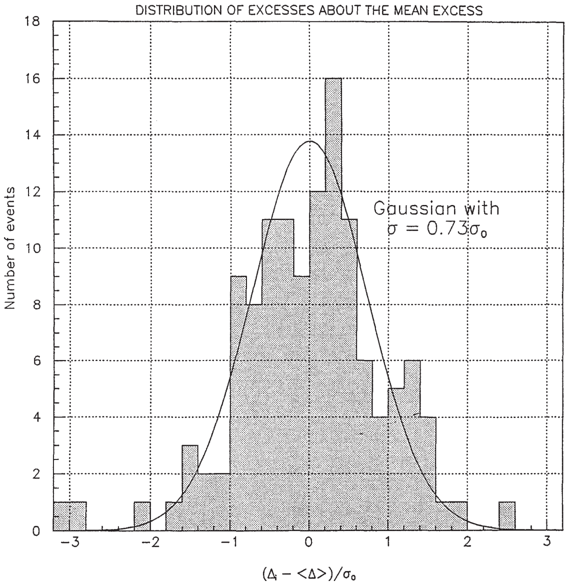


Fig. 4. Distribution of excesses about the mean excess for the maximum curvature method. The agreement with expectation is reassuring.

this result as indicating that

(i) the different zenith angle results are consistent, and our standardisation procedure – viz. displacements of the abscissae – is reasonable;

(ii) our estimates of the uncertainty of the intensities (mainly reading errors) are reasonable.

2.3. Comparison with previous results

Fig. 5 gives a summary of previous work and its comparison with the present analysis. Starting at the top there are the results found by us from an examination of the TUNKA and HEGRA Cerenkov detectors (see [5]). The pattern here is seen to be quite strong.

Next comes the summary by us of the world's data [5], the two sets of data points referring to, separately, near vertical and inclined showers.

Finally, there are the results of the present analysis from Fig. 2a.

The data are arranged in this manner because we expect the amplitude of the pattern to diminish as one goes from (i) the essentially "calorimetric" result from Cerenkov observations to (ii) the average for a region some distance up in the atmosphere (mean atmospheric depth $\simeq$ 818 g cm^{-2} for near vertical showers and 968 g cm^{-2} for the inclined showers) to (iii) the present work which has a mean atmospheric depth of $\simeq$ 960 g cm^{-2}.

It is seen that the amplitudes of the O-peak do decrease from the Cerenkov data to the vertical EAS set (centre box) but the inclined EAS (centre box) is the same as the near vertical EAS (centre box) and higher than the CASA-MIA results at a similar mean depth. There is thus only limited support for the hypothesis from the comparison of amplitudes.

Bearing in mind the identified dependence of the separation from the O-peak on the spectral slope before the knee, the presence of a peak in the region of 0.6 is reassuring.

The extent to which the pattern for CASA-MIA resembles that for our world-survey (viz. the lower two graphs in Fig. 5) can be examined by plotting the amplitudes of Δ against one another (each pair for the same abscissa). The problem of small pattern displacement due to different values of K_1 will cause "noise" but this should not be too serious if there is indeed a definite correlation. Fig. 6 shows the result. It is concluded that the agreement is good; the correlation is significant at the 3σ level.

In [5] we commented on the possible existence of a small peak due to "heavy" nuclei (Ne–S) in between the O- and Fe-peaks. Inspection of Figs. 1a, 1b and 2a shows that there may well be, indeed, such a peak in the CASA-MIA results which is not far from the intermediate peak for the others (Fig. 5).

2.4. Evidence for hydrogen and helium

There could be, in fact there *should* be, hydrogen and helium nuclei accelerated by the local SN. It is well known that where direct measurements of such nuclei have been made, the spectra are steep; nevertheless, flatter spectra from the single source could just be starting to come in.

In the absence of development fluctuations, and for energy spectra represented by delta-functions protons and helium nuclei would manifest themselves by broad peaks at the positions indicated in Fig. 5. In fact, because of development fluctuations (see Section 2.3) and the fact that the spectra will not be delta-functions, the "peaks" will be smeared out considerably and all that will remain will be very broad maxima. Away from the near-vertical, certainly, and pos-

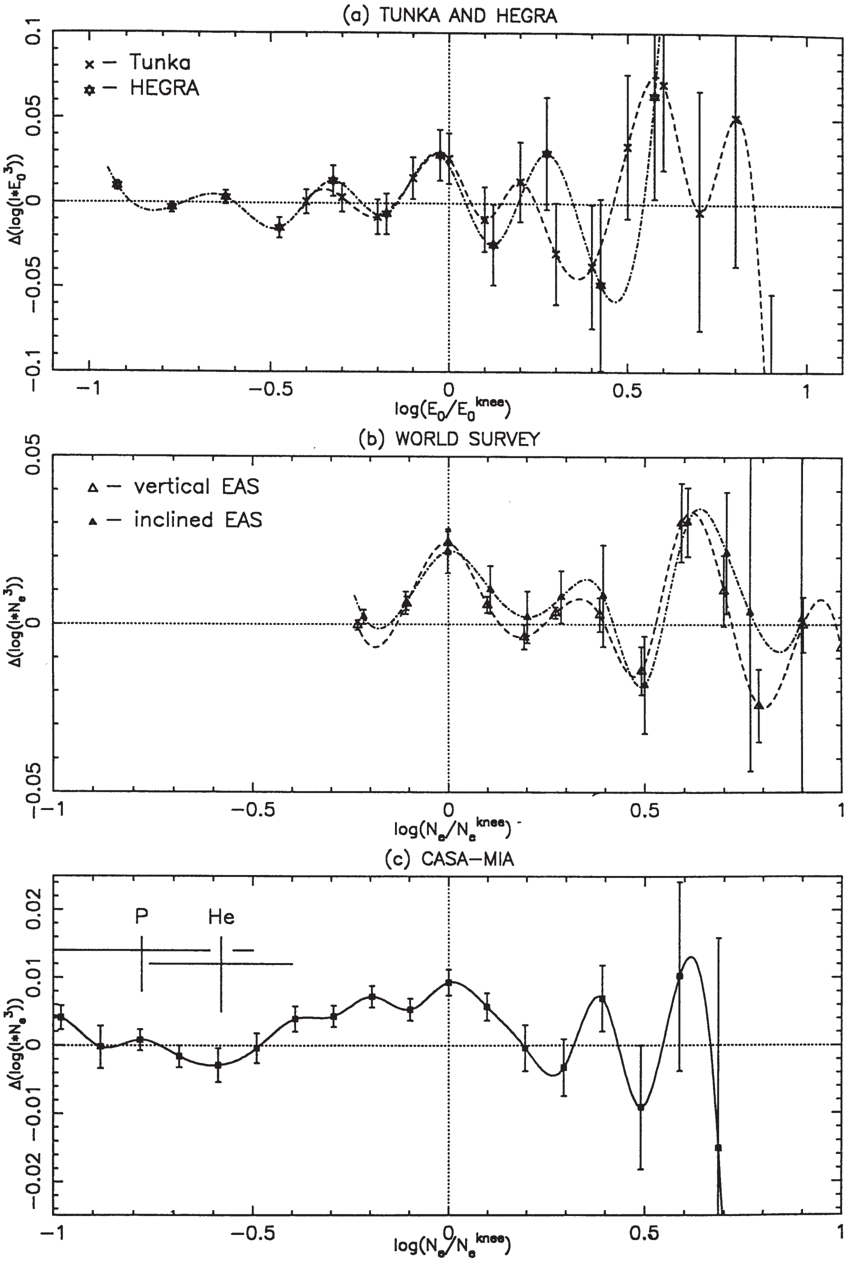


Fig. 5. Comparison of previous work with the present work. (a) Cerenkov data, Tunka and Hegra (see [5]); the scale of the abscissa has been contracted slightly to allow comparison with (b) and (c). (b) Our own "World Survey" [5]. (c) The present analysis of CASA-MIA, integrated over all zenith angles. "P" and "He" indicate the broad regions over which excesses from those particles would occur if their intensities were sufficient. In every case, the lines drawn simply join the points.

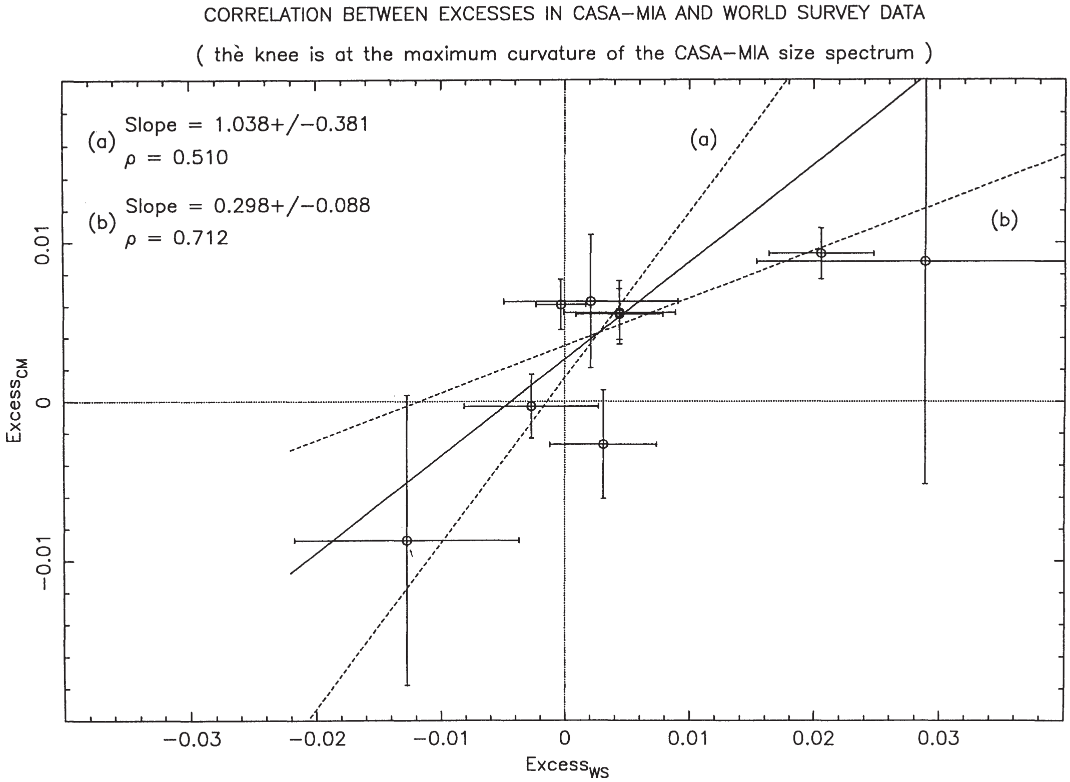


Fig. 6. Correlations between the excesses in our analysis of CASA-MIA, i.e. Fig. 5c, and the "World Survey", for the vertical direction, in Fig. 5b. The two dashed lines represent the best fits for zero errors in, alternatively, abscissa and ordinate. The full line is the bisector. The significance of the fit is seen to be at about the 3σ level.

sibly even for near-vertical showers, P and He will not be recognisable as separate peaks but what might be seen is a general increase of intensity as one proceeds to smaller $\log(N_e/N_e^{\rm knee})$ below about -0.5. Fig. 5 (CASA-MIA) shows no evidence for helium nuclei and only a hint for protons.

Of greater importance is the observation [1] that, if the showers are divided into light (protons plus He-nuclei) and heavy ($Z > 2$), and if the preferred (by us) non-SYBIL interaction model is used, then the light nuclei show a sharp knee. This knee occurs at $\log E_{\rm GeV} \simeq 5.6$ and has sharpness $S_{\rm max} \simeq 2.8 \pm 0.5$, the slope before the knee is 2.4 and above it 3.1. All these features are consistent with the detection of light nuclei from a single source.

3. The mass spectrum

3.1. Our previous work

In a recent paper [7], as mentioned in the introduction, we analysed the world's data on muon-to-electron ratios and on shower depth of maximum, to attempt to solve two problems: what is the best interaction/propagation model to adopt for EAS calculations in the range 10^4–10^8 GeV, and what is the mean logarithmic primary mass ($\langle \ln A \rangle$) as a function of energy?

The answers were: QGSJET [14] and the dependence shown in Fig. 7a.

It is reassuring that our predicted mean mass (from the Single Source model) is close to that derived.

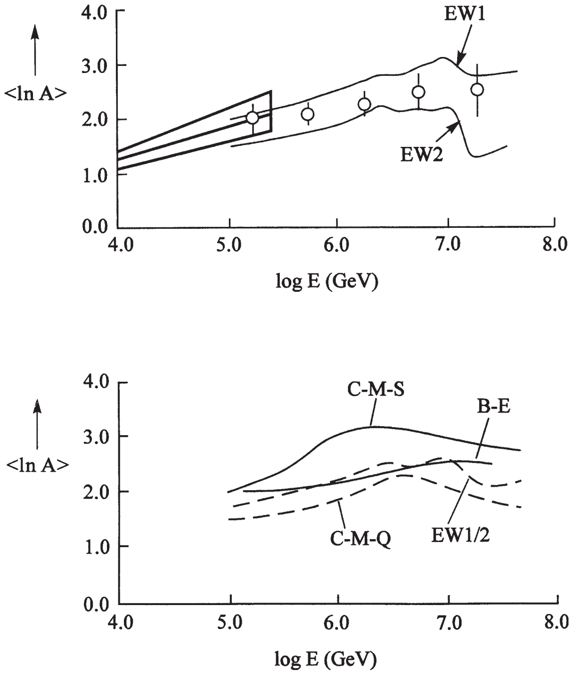


Fig. 7. (a) $\langle \ln A \rangle$ for primary cosmic rays from our earlier work [7]. The "box" relates to direct measurements, EW1 and EW2 are for variants of our single source model and the points are from the comprehensive analysis [7] of the world's EAS data. (b) $\langle \ln A \rangle$ versus energy for various situations: B-E: Best estimate from the points given in Fig. 5. EW1/2: mean of EW1 and EW2, i.e. our Single Source Model prediction. C-M-S: Estimate from CASA-MIA results as given by them for their model (SIBYLL). C-M-Q: Results for CASA-MIA converted, approximately, by us to correspond to use of the QGSJET model.

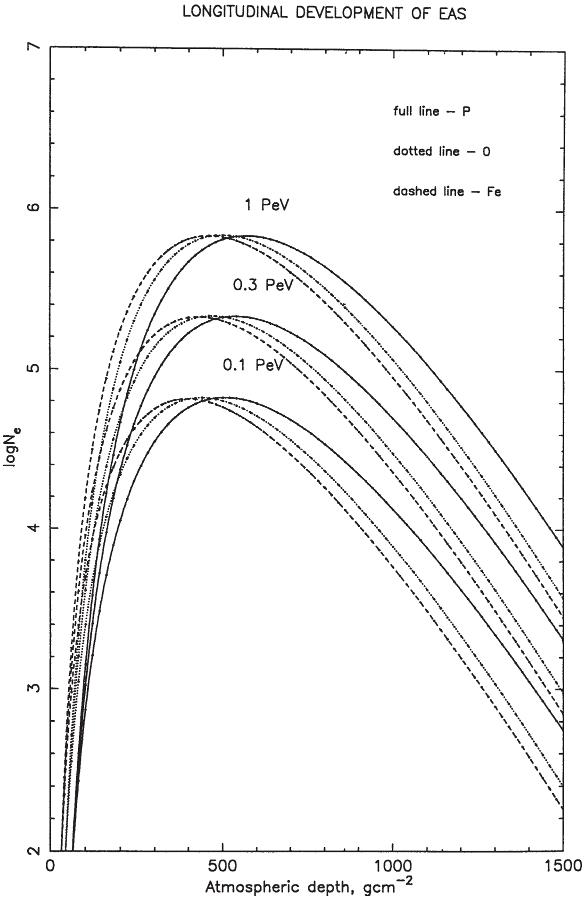


Fig. 8. Longitudinal development of EAS, as computed by us.

3.2. The CASA-MIA results

In [1] a detailed description is given of the technique used to endeavour to determine particle masses, on a statistical basis. The quantity determined was the "corrected average fraction of proton nearest neighbours" versus size (we refer to this quantity as f_p). The authors also give their derived f_p values versus their estimate of the primary energy. It appears to us that there is probably near linearity between $1 - f_p$ and $\ln A$, specifically: $\ln A = 4(1 - f_p)$. The reason for this suggestion is that in studies of depth of maximum, and muon-to-electron ratios, a similar situation prevails. (Professor Jim Matthews, of the CASA-MIA project, told us that he has arrived at essentially the same conclusion via a different route). The dependence of $\langle \ln A \rangle$ on primary energy derived by us in this way is given in Fig. 7b (denoted C-M-S).

It should be noted that the CASA-MIA group adopted the SIBYLL interaction model [15]. Our contention in [7] was that the QGSJET model is to be preferred. A reworking of CASA-MIA results along the lines discussed in [7] gives the line marked C-M-Q in Fig. 7b. This, then, is to be compared with our own estimates. Although not coincident, the estimates are close.

4. Conclusions

The CASA-MIA results certainly show evidence for structure in the size spectra if the quoted results are as precise as the statistical accuracy claims. The structure is rather similar to what we have found from analysis of the world's EAS data and from the Cerenkov experiments, but the amplitude of the effect is rather smaller.

The CASA-MIA analysis of the mass composition supports our contention about the dependence of mean mass on primary energy rather strongly, when the same interaction model is used in both cases.

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