BARIÕES

Momento angular orbital

dois nºs quânticos ℓ , ℓ'

estado fundamental $\ell = \ell' = 0$

Spin

3 quarks de spin ½ $\Rightarrow$ 8 combinações possíveis
$(\uparrow\uparrow\uparrow, \uparrow\uparrow\downarrow, \uparrow\downarrow\uparrow, \ldots)$

2 spins possíveis $J = 3/2$
$J = 1/2$

8 estados próprios de spin

$$\left.\begin{array}{l} |\tfrac{3}{2}\,\tfrac{3}{2}\rangle = (\uparrow\uparrow\uparrow) \\ |\tfrac{3}{2}\,\tfrac{1}{2}\rangle = (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)/\sqrt{3} \\ |\tfrac{3}{2}\,{-\tfrac{1}{2}}\rangle = (\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow)/\sqrt{3} \\ |\tfrac{3}{2}\,{-\tfrac{3}{2}}\rangle = (\downarrow\downarrow\downarrow) \end{array}\right\} \psi_s$$

Spin 3/2

completamente simétrica

$$\left.\begin{array}{l} |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_{12} = (\uparrow\downarrow - \downarrow\uparrow)\uparrow/\sqrt{2} \\ |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle_{12} = (\uparrow\downarrow - \downarrow\uparrow)\downarrow/\sqrt{2} \end{array}\right\} \psi_{12}$$

spin 1/2

antisimétrica em troca 1-2

$$\left.\begin{array}{l} |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_{23} = \uparrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2} \\ |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle_{23} = \downarrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2} \end{array}\right\} \psi_{23}$$

antisimétrica em troca 2-3

$$\psi_{13} = \psi_{12} + \psi_{23} \Rightarrow \begin{array}{l} |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_{13} = (\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)/\sqrt{2} \\ |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle_{13} = (\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow)/\sqrt{2} \end{array}$$

BARIÕES : estados de 3 partículas (quarks) idênticas

⇒ FUNÇÃO DE ONDA ANTISIMÉTRICA

$$\psi_{\text{barião}} = \psi(\text{space})\,\psi(\text{spin})\,\psi(\text{flavor})\,\psi(\text{color})$$

1) $\ell = \ell' = 0$ ⇒ $\psi(\text{space})$ simétrica

2) SPIN : $\psi(\text{spin})$ simétrica $J = 3/2$

$\psi(\text{spin})$ mixta $J = 1/2$

3) FLAVOR $SU(3)_f$: $3^3 = 27$ combinações $uuu, uud, udu, \dots$

10 estados ψ_s completamente simétricos

8 estados ψ_{12} antisimétricos em 1–2

8 estados ψ_{23} antisimétricos em 2–3

1 estado ψ_a completamente antisimétrico

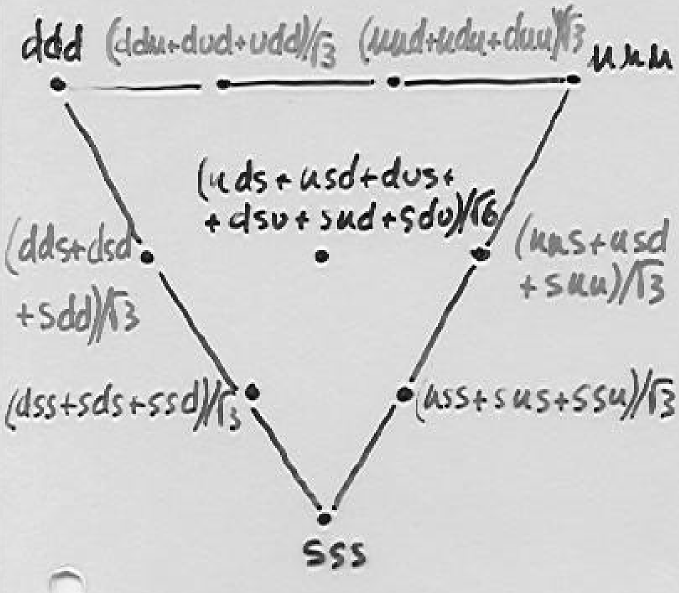
$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

4) COR $SU(3)_c$: 3 cores r, g, b

⇒ PARTÍCULAS NA NATUREZA SÓ APARECEM NO ESTADO SINGLETO (ANTISIMÉTRICO)

$$\psi_a(\text{cor}) = (rgb - rbg + gbr - grb + brg - bgr)/\sqrt{6}$$

$\boxed{\psi_S\text{: ESTADOS COMPLETAMENTE SIMÉTRICOS}}$



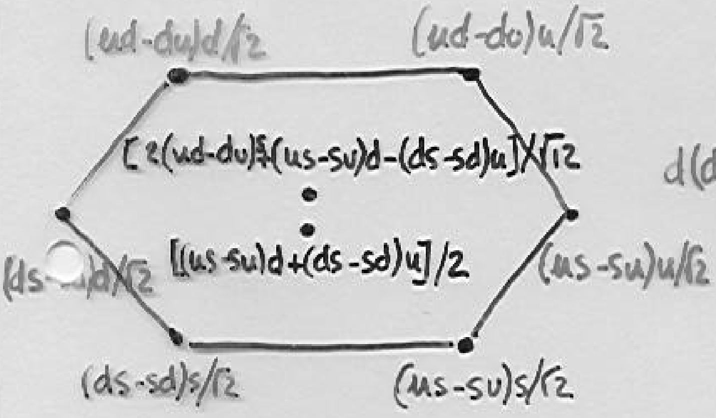
DECUPLETO

$\boxed{\psi_A\text{: ESTADO COMPLETAMENTE ANTISIMÉTRICO}}$

$$(uds-usd+dsu-dus+sud-sdu)/\sqrt{6}$$

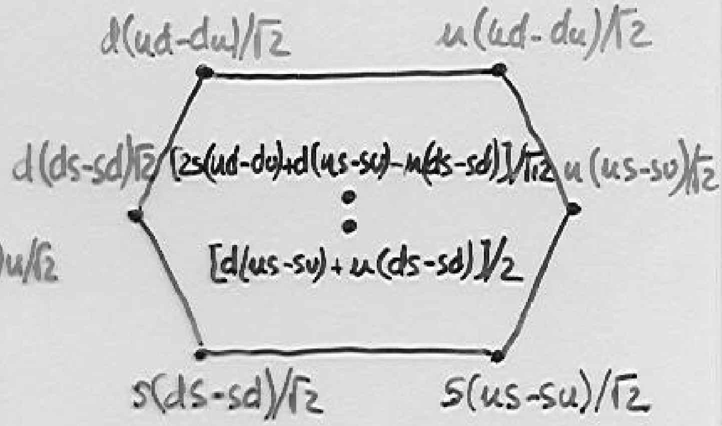
SINGLETO

$\boxed{\psi_{12}\text{: ANTISIMÉTRICO EM 1-2}}$



OCTETO

$\boxed{\psi_{23}\text{: ANTISIMÉTRICO EM 2-3}}$



OCTETO

$q_1 q_1 q_1 \rightarrow$ 1 combinação SIMÉTRICA $\rightarrow$ DECUPLETO

$q_1 q_2 q_2 \rightarrow$ 3 combinações $\begin{cases} \text{1 SIMÉTRICA} \rightarrow \text{DECUPLETO} \\ \text{2 MISTA} \rightarrow \text{OCTETOS} \end{cases}$

$q_1 q_2 q_3 \rightarrow$ 6 combinações $\begin{cases} \text{1 SIMÉTRICA} \rightarrow \text{DECUPLETO} \\ \text{4 MISTAS} \rightarrow \text{OCTETOS} \\ \text{1 ANTISIMÉTRICA} \rightarrow \text{SINGLETO} \end{cases}$

MOMENTOS MAGNÉTICOS DOS BARIÕES

Momento magnético de uma partícula de spin 1/2 :

$$\vec{\mu} = \frac{q}{mc}\vec{S}$$

$$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c}$$

$$\mu_d = -\frac{1}{3}\frac{e\hbar}{2m_d c}$$

$$\mu_s = -\frac{1}{3}\frac{e\hbar}{2m_s c}$$

amplitude (segundo z) : $\mu = \frac{q\hbar}{2mc}$

Momento magnético dos bariões :

$$\mu_B = \langle B\uparrow | (\mu_1 + \mu_2 + \mu_3) | B\uparrow \rangle$$

Aplicação ao protão:

1º termo da função de onda $\frac{2}{3\sqrt{2}} [u(\uparrow) u(\uparrow) d(\downarrow)]$

$$(\mu_1 + \mu_2 + \mu_3) | u(\uparrow) u(\uparrow) d(\downarrow) \rangle = (\mu_u + \mu_u - \mu_d) | u(\uparrow) u(\uparrow) d(\downarrow) \rangle$$

$$\Rightarrow \left(\frac{2}{3\sqrt{2}}\right)^2 (2\mu_u - \mu_d) = \frac{2}{9}(2\mu_u - \mu_d)$$

de forma análoga para os outros termos :

$$\mu_p = 3\left[\frac{2}{9}(2\mu_u - \mu_d) + \frac{1}{18}\mu_d + \frac{1}{18}\mu_d\right] = \frac{1}{3}(4\mu_u - \mu_d)$$

MOMENTOS MAGNÉTICOS - RESULTADOS

Barião	Momento	Experiência	Modelo
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	2.793	2.79
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	−1.913	−1.86
Λ	μ_s	−0.61	−.58
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	2.33±.13	2.68
Σ^0	$\frac{2}{3}(\mu_u+\mu_d) - \frac{1}{3}\mu_s$		0.82
Σ^-	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	−1.41±.25	−1.05
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	−1.253±.014	−1.40
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	−.69±.04	−0.47

Valores em múltiplos do magnetão nuclear $\frac{e\hbar}{2m_pc}$

m_p - massa do protão

FUNÇÕES DE ONDA DOS BARIÕES

DECUPLETO : $\psi_{decupleto} = \psi_S(\text{spin})\ \psi_S(\text{flavor}) \cdot \psi_a(\text{cor})$

$J = 3/2$ decupleto $SU(3)_f$ singleto $SU(3)_c$

OCTETO : $\psi_{octeto} = \frac{\sqrt{2}}{3}\left[\psi_{12}(\text{spin})\psi_{12}(\text{flavor}) + \psi_{23}(\text{spin})\psi_{23}(\text{flavor}) + \psi_{13}(\text{spin})\psi_{13}(\text{flavor})\right] \cdot \psi_a(\text{cor})$

Função de onda do protão, spin up

$$|p\uparrow\rangle = \frac{\sqrt{2}}{3}\left[\frac{1}{2}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)(udu - duu) + \frac{1}{2}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(uud - udu) + \frac{1}{2}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(uud - duu)\right] =$$

$$= \frac{1}{3\sqrt{2}}\left[uud\,(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + udu\,(2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + duu\,(2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow)\right]$$

$$= \frac{2}{3\sqrt{2}}\,u(\uparrow)u(\uparrow)d(\downarrow) - \frac{1}{3\sqrt{2}}\,u(\uparrow)u(\downarrow)d(\uparrow) - \frac{1}{3\sqrt{2}}\,u(\downarrow)u(\uparrow)d(\uparrow)$$

+ permutações

MASSA DOS BARIÕES

$$M(\text{barião}) = m_1 + m_2 + m_3 + A' \left[\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2} + \frac{\vec{S}_1 \cdot \vec{S}_3}{m_1 m_3} + \frac{\vec{S}_2 \cdot \vec{S}_3}{m_2 m_3} \right]$$

1) $J^2 = (\vec{S}_1 + \vec{S}_2 + \vec{S}_3)^2 = S_1^2 + S_2^2 + S_3^2 + 2(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_1 \cdot \vec{S}_3 + \vec{S}_2 \cdot \vec{S}_3)$

$$\Rightarrow \vec{S}_1 \cdot \vec{S}_2 + \vec{S}_1 \cdot \vec{S}_3 + \vec{S}_2 \cdot \vec{S}_3 = \frac{\hbar^2}{2}\left[j(j+1) - \frac{9}{4} \right] =$$

$$\begin{cases} = \frac{3}{4}\hbar^2 & j = 3/2 \quad \text{decupleto} \\ = -\frac{3}{4}\hbar^2 & j = 1/2 \quad \text{octeto} \end{cases}$$

$m_u = m_d$

$$\left.\begin{array}{ll} \text{Nucleão :} & M_N = 3m_u - \frac{3}{4}\frac{\hbar^2}{m_u^2} A' \\ \Delta \quad : & M_\Delta = 3m_u + \frac{3}{4}\frac{\hbar^2}{m_u^2} A' \\ \Omega^- \quad : & M_{\Omega^-} = 3m_s + \frac{3}{4}\frac{\hbar^2}{m_s^2} A' \end{array}\right\} \begin{array}{c} \text{Casos} \\ m_1 = m_2 = m_3 \end{array}$$

- NO DECUPLETO $(j=3/2)$ TODOS OS PARES FORMAM SPIN 1 :

$i,k = 1,2,3$ $$(\vec{S}_i + \vec{S}_k)^2 = S_i^2 + S_k^2 + 2\,\vec{S}_i\cdot\vec{S}_k = 2\hbar^2$$

$$\Rightarrow \vec{S}_1\cdot\vec{S}_2 = \vec{S}_1\cdot\vec{S}_3 = \vec{S}_2\cdot\vec{S}_3 = \frac{\hbar^2}{4}$$

donde:

$$M_{\Sigma^*} = 2m_u + m_s + \frac{\hbar^2}{4}A'\left[\frac{1}{m_u^2} + \frac{2}{m_u m_s}\right]$$

$$M_{\Xi^*} = m_u + 2m_s + \frac{\hbar^2}{4}A'\left[\frac{2}{m_u m_s} + \frac{1}{m_s^2}\right]$$

- NO OCTETO $(j=1/2)$, o caso do Σ e Λ :

$$\left.\begin{array}{l} \Sigma : \text{ isospin } 1 \Rightarrow \text{spin}_{u\text{-}d}\ 1 \\ \Lambda : \text{ isospin } 0 \Rightarrow \text{spin}_{u\text{-}d}\ 0 \end{array}\right\} \Rightarrow \psi_{\text{spin/flavor}} \text{ simétrica}$$

$$\Sigma : (\vec{S}_u + \vec{S}_d)^2 = S_u^2 + S_d^2 + 2\vec{S}_u\cdot\vec{S}_d = 2\hbar^2 \Rightarrow \vec{S}_u\cdot\vec{S}_d = \frac{\hbar^2}{4}$$

$$\Lambda : (\vec{S}_u + \vec{S}_d)^2 = 0 \Rightarrow \vec{S}_u\cdot\vec{S}_d = -\frac{3}{4}\hbar^2$$

donde:

$$M_\Sigma = 2m_u + m_s + A'\left[\frac{\vec{S}_u\cdot\vec{S}_d}{m_u m_d} + \frac{(\vec{S}_1\cdot\vec{S}_2 + \vec{S}_1\cdot\vec{S}_3 + \vec{S}_2\cdot\vec{S}_3 - \vec{S}_u\cdot\vec{S}_d)}{m_u m_s}\right]$$

$$= 2m_u + m_s + \frac{\hbar^2}{4}A'\left(\frac{1}{m_u^2} - \frac{4}{m_u m_s}\right)$$

MASSA DOS BARIÕES – RESULTADOS

Barião	Experimental	Modelo	(MeV/c^2)
N	939	939	
Λ	1116	1114	
Σ	1179	1193	
Ξ	1327	1318	
Δ	1239	1232	
Σ^*	1381	1384	
Ξ^*	1529	1533	
Ω	1682	1672	

Com $m_u = m_d = 363 \text{ MeV}/c^2$

$m_s = 538 \text{ MeV}/c^2$

$A' = (2 m_u/\hbar)^2 \cdot 50 \text{ MeV}/c^2$