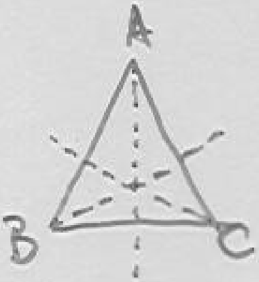
SIMETRIAS

SIMETRIA: operação sobre um sistema que o deixa INVARIANTE



Ex: operações de simetria
$R^+ \; R^- \; R_a \; R_b \; R_c \; I$

Propriedades:

1) Fecho : $R_i \cdot R_j$ pertence ao conjunto

2) Identidade : $I . R_i = R_i \cdot I = R_i$

3) Inverso : $R_i R_i^{-1} = R_i^{-1} R_i = I$

4) Associatividade: $R_i (R_j R_k) = (R_i R_j) R_k$

$\Rightarrow$ GRUPO DE SIMETRIA

GRUPOS
- Abelianos $R_i R_j = R_j R_i$
- Não Abelianos $R_i R_j \neq R_j R_i$

GRUPOS finitos e infinitos
contínuos e discretos

Referência:
'Introduction to Elementary Particles', D. Griffiths, J. Wiley and Sons

GRUPOS DE MATRIZES

$U(n)$: colecção das matrizes unitárias $n \times n$ $(U^{-1} = \tilde{U}^*)$

$SU(n)$: matrizes unitárias com $\det = 1$

$O(n)$: matrizes unitárias reais $(O^{-1} = \tilde{O})$
(ortogonais)

$SO(n)$: matrizes ortogonais com $\det = 1$
↳ grupo das rotações em n-dimensões

Um grupo de operações pode ser REPRESENTADO por um grupo de matrizes :

$$a \cdot b = c \implies M_a \cdot M_b = M_c$$

> REPRESENTAÇÃO FUNDAMENTAL e REPRESENTAÇÕES DE DIMENSÃO SUPERIOR

> REPRESENTAÇÕES REDUTÍVEIS $M_a = \begin{pmatrix} M_a^1 & 0 \\ 0 & M_a^2 \end{pmatrix}$

SPIN E MOMENTO ORBITAL ANGULAR

Quantificação:

orbital: $L^2 = \ell(\ell+1)\hbar$ $\qquad \ell = 0, 1, 2, \ldots$

$L_z = m_\ell \hbar$ $\qquad m_\ell = -\ell, -\ell+1, \ldots, \ell-1, \ell$

Spin $S^2 = s(s+1)\hbar$ $\qquad s = 0, 1/2, 1, 3/2, 2 \ldots$

$S_z = m_s \hbar$ $\qquad m_s = -s, \ldots, +s$

$s=0$ $\quad \pi, K, \ldots$

$s=1/2$ $\quad e, p, n, q, \ldots$

$s=1$ $\quad \rho, \psi, \gamma, g, \ldots$

$s=3/2$ $\quad \Delta, \Omega^-, \ldots$

ADIÇÃO DE MOMENTO ANGULAR $\quad J = J_1 + J_2$

$|j_1 m_1\rangle + |j_2 m_2\rangle \Rightarrow$ j one of $|j_1-j_2|, \ldots, |j_1+j_2|$

$m = m_1 + m_2$

$$|j_1 m_1\rangle |j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{j_1+j_2} C^{j\, j_1\, j_2}_{m\, m_1\, m_2} |j\, m\rangle$$

$\downarrow$

COEFICIENTES DE CLEBSCH-GORDON

C^2: probabilidade de obter $j(j+1)\hbar^2$ na medição de J^2.

MESÕES $(q\bar{q})$

$j = 1/2 + 1/2 = 1$ ou $j = 1/2 - 1/2 = 0$

$\downarrow$ mesões vector: π's, K's, η, η'

$\downarrow$ Scalar mesons: ρ's, K^*'s, ϕ, ω

BARIÕES (qqq)

$1 + 1/2 = 3/2$

$1 - 1/2 = 1/2 \quad \Rightarrow$ spin $3/2$ ou $1/2$

$0 + 1/2 = 1/2$

Combinação de dois estados de spin 1/2

$1/2 \times 1/2$	1, +1	1, 0	0, 0	1, −1
+1/2 +1/2	1			
+1/2 −1/2		1/2	1/2	
−1/2 +1/2		1/2	−1/2	
−1/2 −1/2				1

$u = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$d = |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$

$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = |1\,1\rangle$

$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \left(\tfrac{1}{\sqrt{2}}\right)|1\,0\rangle + \left(\tfrac{1}{\sqrt{2}}\right)|0\,0\rangle$

$|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \left(\tfrac{1}{\sqrt{2}}\right)|1\,0\rangle - \left(\tfrac{1}{\sqrt{2}}\right)|0\,0\rangle$

$|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = |1\,-1\rangle$

Tripleto:

$|1\,1\rangle = |u\rangle|u\rangle$

$|1\,0\rangle = \tfrac{1}{\sqrt{2}}\left[|u\rangle|d\rangle + |d\rangle|u\rangle\right]$

$|1\,-1\rangle = |d\rangle|d\rangle$

Singleto:

$|0\,0\rangle = \tfrac{1}{\sqrt{2}}\left[|u\rangle|d\rangle - |d\rangle|u\rangle\right]$

SPIN 1/2

Spinors $\quad |1/2 \ 1/2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1/2 \ -1/2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Estado geral $\quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\alpha^2| + |\beta^2| = 1$$

Operadores $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$:

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ex: $\hat{S}_x$

valores próprios $\pm \hbar/2$

vectores próprios $\chi_\pm = \begin{pmatrix} 1/\sqrt{2} \\ \pm 1/\sqrt{2} \end{pmatrix}$

vector arbitrário $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = a \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + b \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

$$\Rightarrow a = \frac{1}{\sqrt{2}}(\alpha + \beta) \qquad b = \frac{1}{\sqrt{2}}(\alpha - \beta)$$

Matrizes de Pauli:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S} = (\hbar/2)\, \sigma$$

ROTAÇÃO DE SPINORS

$$\begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = U(\theta) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

com $U(\theta) = e^{-i\vec{\theta}.\vec{\sigma}/2}$

$\vec{\theta}$: vector segundo o eixo de rotação e de amplitude θ

$$e^A \equiv 1 + A + \frac{1}{2} A^2 + \frac{1}{3!} A^3 + \cdots$$

U - matrizes unitárias com det 1 $\Rightarrow SU(2)$

SIMETRIA DE ISOSPIN

Nucleão : $N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

protão $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1/2\ 1/2\rangle$ neutrão $n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|1/2\ -1/2\rangle$

$\Rightarrow$ ISOSPIN $|I, I_3\rangle$

AS INTERACÇÕES FORTES SÃO INVARIANTES FACE A ROTAÇÕES NO ESPAÇO DE ISOSPIN

$\Rightarrow$ CONSERVAÇÃO DE ISOSPIN

π $I=1$ $\pi^+ = |1\ 1\rangle$ $\pi^0 = |1\ 0\rangle$ $\pi^- = |1\ -1\rangle$

Λ $I=0$ $\Lambda = |0\ 0\rangle$

Δ $I=3/2$ $\Delta^{++} = |3/2\ 3/2\rangle$ $\Delta^+ = |3/2\ 1/2\rangle$ $\Delta^0 = |\frac{3}{2}\ -\frac{1}{2}\rangle$ $\Delta^- = |\frac{3}{2}\ -\frac{3}{2}\rangle$

$Q = I_3 + \frac{1}{2}(B + S)$

↳ estranheza

↳ nº bariónico

↳ 3ª comp. isospin

SISTEMA DE DOIS NUCLEÕES (DEUTRÃO)

$$I^1 = 1/2 \quad I^2 = 1/2 \quad \Rightarrow \quad I = 1 \text{ ou } 0$$

TRIPLETO

$$|1\,1\rangle = pp$$
$$|1\,0\rangle = \frac{1}{\sqrt{2}}(pn + np)$$
$$|1\,{-1}\rangle = nn$$

SINGLETO $\quad |0,0\rangle = \frac{1}{\sqrt{2}}(pn - np)$

Na natureza não existem estados ligados pp ou nn

$\Rightarrow$ deutrão $\equiv$ singleto

hipótese: potencial de interacção NN contem termo da forma $\vec{I}^1 \cdot \vec{I}^2$

$$\vec{I}^{(1)} \cdot \vec{I}^{(2)} = -3/4 \quad \text{singleto}$$
$$\vec{I}^{(1)} \cdot \vec{I}^{(2)} = 1/4 \quad \text{tripleto}$$

DIFUSÃO NUCLEÃO - NUCLEÃO

(a) $p + p \to d + \pi^+$

(b) $p + n \to d + \pi^0$

(c) $n + n \to d + \pi^-$

Estados de isospin: (deutrão $I=0$)

esquerda		direita
$\|11\rangle$	a	$\|11\rangle$
$\frac{1}{\sqrt{2}}(\|10\rangle + \|00\rangle)$	b	$\|10\rangle$
$\|1-1\rangle$	c	$\|1-1\rangle$

Conservação de isospin $\Longrightarrow$ $\mathcal{M}_a : \mathcal{M}_b : \mathcal{M}_c = 1 : \frac{1}{\sqrt{2}} : 1$

$\sigma_a : \sigma_b : \sigma_c = 2 : 1 : 2$

DIFUSÃO PIÃO-NUCLEÃO $\pi N \to \pi N$

Processos elásticos:

(a) $\pi^+ p \to \pi^+ p$
(b) $\pi^0 p \to \pi^0 p$
(c) $\pi^- p \to \pi^- p$
(d) $\pi^+ n \to \pi^+ n$
(e) $\pi^0 n \to \pi^0 n$
(f) $\pi^- n \to \pi^- n$

Processos de troca de carga:

(g) $\pi^+ n \to \pi^0 p$
(h) $\pi^0 p \to \pi^+ n$
(i) $\pi^0 n \to \pi^- p$
(j) $\pi^- p \to \pi^0 n$

$\pi : I=1 \quad N : I=1/2 \quad \Rightarrow I_{tot} = 3/2$ ou $1/2$

$\pi^+ p : |1\,1\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle = |\tfrac{3}{2}\,\tfrac{3}{2}\rangle$

$\pi^0 p : |1\,0\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \sqrt{2/3}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle - (1/\sqrt{3})\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$\pi^- p : |1\,{-1}\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle = (1/\sqrt{3})\,|\tfrac{3}{2}\,{-\tfrac{1}{2}}\rangle - \sqrt{2/3}\,|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle$

$\pi^+ n : |1\,1\rangle |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle = (1/\sqrt{3})\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{2/3}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$\pi^0 n : |1\,0\rangle |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle = \sqrt{2/3}\,|\tfrac{3}{2}\,{-\tfrac{1}{2}}\rangle + (1/\sqrt{3})\,|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle$

$\pi^- n : |1\,{-1}\rangle |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle = |\tfrac{3}{2}\,{-\tfrac{3}{2}}\rangle$

$\mathcal{M}_3$: amplitude de reacções com $I=3/2$
$\mathcal{M}_1$: amplitude de reacções com $I=1/2$

Resultados:

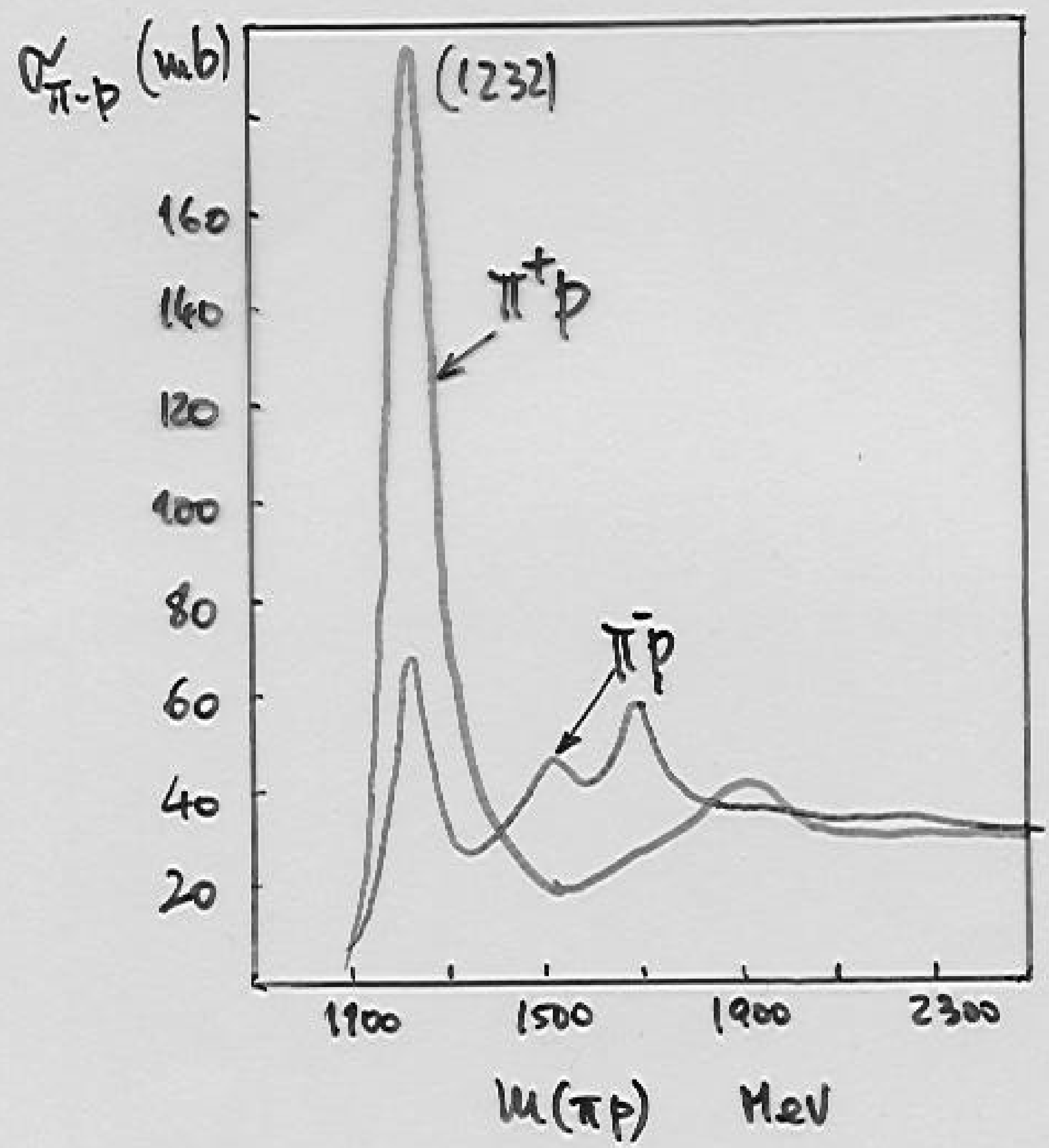
$$\mathcal{M}_a = \mathcal{M}_f = \mathcal{M}_3 \qquad \mathcal{M}_j = \frac{\sqrt{2}}{3}\mathcal{M}_3 - \frac{\sqrt{2}}{3}\mathcal{M}_1$$

$$\mathcal{M}_c = \frac{1}{3}\mathcal{M}_3 + \frac{2}{3}\mathcal{M}_1 \qquad \ldots$$

$$\sigma_a : \sigma_c : \sigma_j = 9|\mathcal{M}_3|^2 : |\mathcal{M}_3 + 2\mathcal{M}_1|^2 : 2|\mathcal{M}_3 - \mathcal{M}_1|^2$$

$\sqrt{s} = 1232$ MeV $\quad \pi N \to \Delta \to \pi N \quad \Delta : I = 3/2 \Rightarrow \mathcal{M}_3 \gg \mathcal{M}_1$

$$\Rightarrow \sigma_a : \sigma_c : \sigma_j = 9 : 1 : 2 \quad \Rightarrow \frac{\sigma_{tot}(\pi^+ p)}{\sigma_{tot}(\pi^- p)} = 3$$



σ π-p (mb)
(1232)
160
140
120
100
80
60
40
20
π+p
π-p
1100
1500
1900
2300
m(πp) MeV