

ESTRUTURA DOS HADRÕES

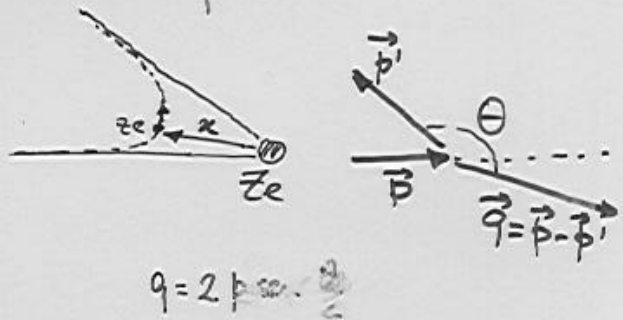
1. DIFUSÃO E FACTORES DE FORMA

Na aproximação de Born :

$$\frac{d\sigma}{d\Omega} = |f(\vec{q})|^2$$

$f(\vec{q})$: amplitude de difusão

$$\vec{q} = \vec{p} - \vec{p}'$$



$$f(\vec{q}) = -\frac{m}{2\pi\hbar^2} \int V(\vec{x}) e^{i\vec{q}\cdot\vec{x}/\hbar} d^3x$$

$V(\vec{x})$: potencial de interação

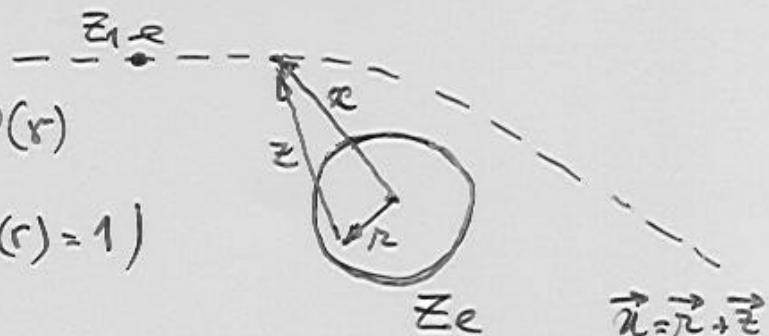
$$V(x) = \frac{g_1 g_2}{x} \rightarrow f(q^2) = -\frac{2m g_1 g_2}{q^2}$$

$$\Rightarrow \left. \frac{d\sigma}{d\Omega} \right|_{\text{Born}} = \frac{4m^2 (g_1 g_2)^2}{q^4}$$

CASO DE PARTÍCULAS ALVO COM ESTRUTURA:

Densidade de carga $\rho(r)$

(normalização: $\int d^3r \rho(r) = 1$)



Potencial: $V(x) = Z_1 Z e^2 \int d^3r \frac{\rho(r)}{x}$

Cálculo de $f(\vec{q})$:

• integral em $\frac{e^{i\vec{q} \cdot \vec{z}/\hbar}}{z}$ (a \vec{r} fixo) $\rightarrow \propto \frac{1}{q^2}$

• integral em $e^{i\vec{q} \cdot \vec{r}/\hbar}$: $F(q^2) = \int d^3r \rho(r) e^{i\vec{q} \cdot \vec{r}/\hbar}$
(transformada de Fourier)

$\Rightarrow \frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Ruth}} \cdot |F(q^2)|^2$

$F(q^2)$: FACTOR DE FORMA

Densidade de probabilidade

Factor de forma

$$\delta(r)$$

$$\rho_0 e^{-r/a}$$

$$\rho_0 e^{-(r/b)^2}$$

$$\left. \begin{array}{l} \rho_0 \quad r \leq R \\ 0 \quad r > R \end{array} \right\}$$

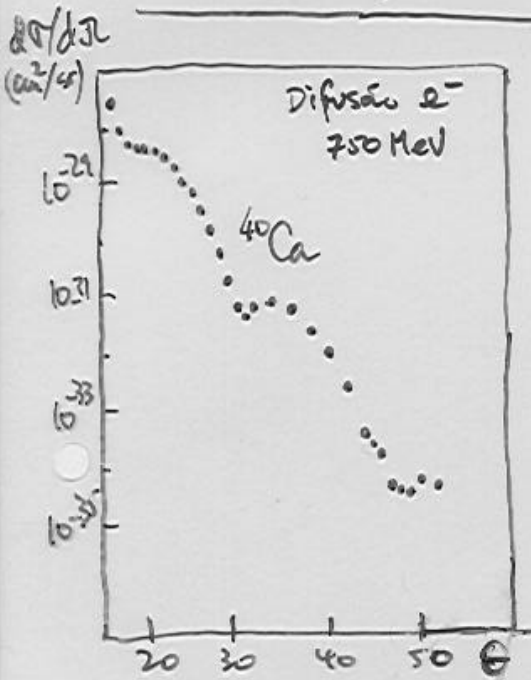
$$1$$

$$(1 + q^2 a^2 / \hbar^2)^{-2}$$

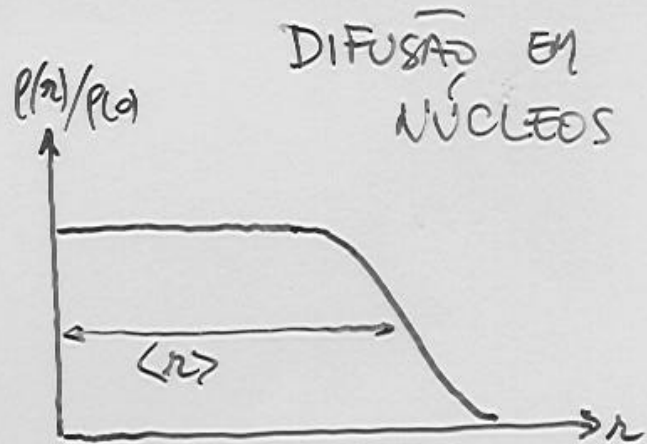
$$e^{-q^2 b^2 / 4 \hbar^2}$$

$$3 \left[\text{sen}(qR/\hbar) - (qR/\hbar) \text{co}(qR/\hbar) \right]$$

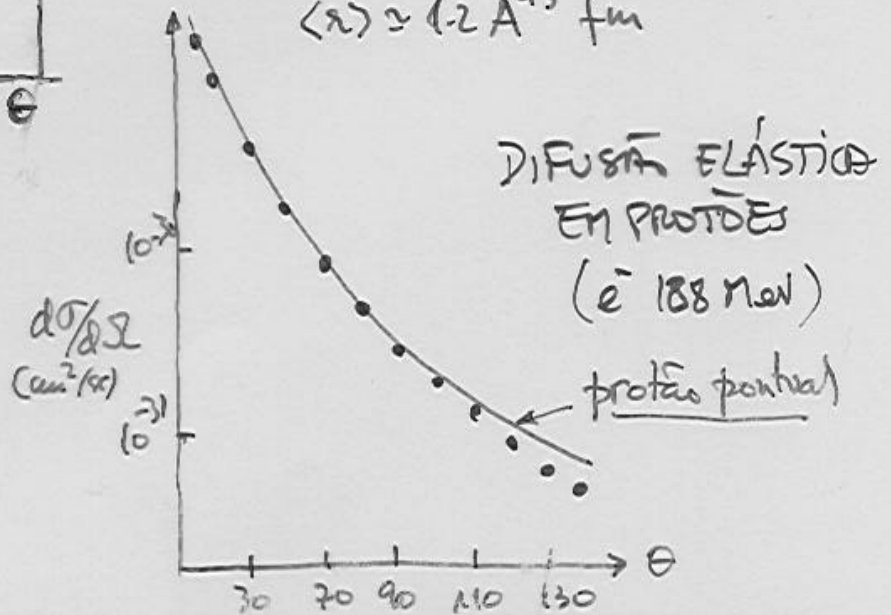
$$(qR/\hbar)^3$$



⇒



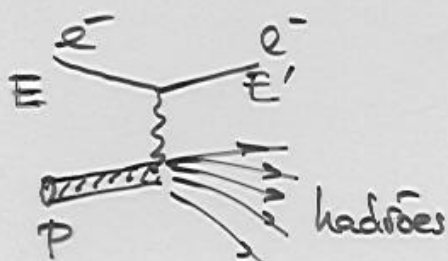
$$\langle r \rangle \approx 1.2 A^{2/3} \text{ fm}$$



DIFUSÃO INELÁSTICA $e-p$:

Dois variáveis cinemáticas : θ

$$\gamma = E - E'$$



Interpretação em termos de factores de forma ?

As 'boas' variáveis :

$$q^2 = \gamma^2 - (\vec{p} - \vec{p}')^2 \quad (\text{quadri-momento transferido})$$

$$W^2 = E_h^2 - p_h^2$$

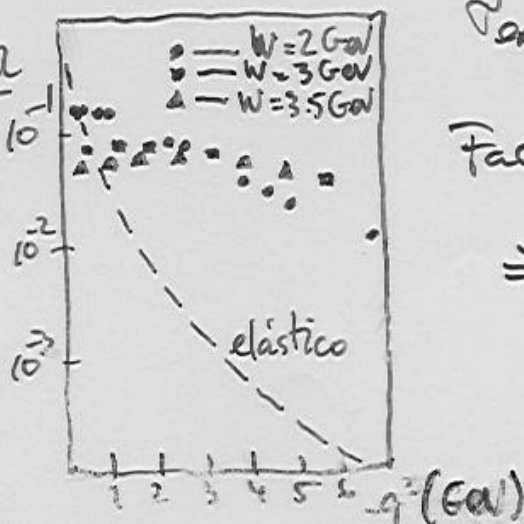
$$E_h = \gamma + m_p$$

$$\vec{p}_h = \vec{p} - \vec{p}'$$

(massa invariante do sistema hadrónico)



$\frac{d^2\sigma/d\Omega dQ^2}{\sigma_{Mott}}$



$$\sigma_{exp}/\sigma_{Mott} = \text{Factor forma}$$

Factor de forma cst

\Rightarrow Alvo pontual

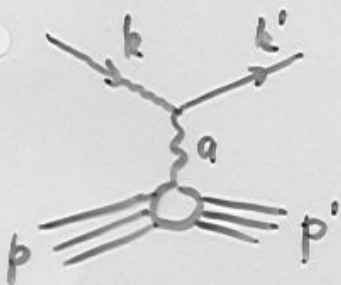
σ = observação experimental dos quarks

2. Difusão electrão-protão

Difusão de electrão sobre partículas do Dirac:

$$\frac{d\sigma}{d\Omega} \Big|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \right) \frac{E'}{E} \left\{ \cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}$$

Difusão sobre o protão



$$T_{fi} = -i \int j_{\mu} \left(-\frac{1}{q^2} \right) J^{\mu} d^4x$$

$$j_{\mu} = -e \bar{u}(k') \gamma^{\mu} u(k) e^{i(k'-k) \cdot x}$$

$$J_{\mu} = e \bar{u}(p') [\dots] u(p) e^{i(p'-p) \cdot x}$$

forma geral: $[\dots] = F_1(q^2) \gamma^{\mu} + \frac{\kappa}{2M} F_2(q^2) i \sigma^{\mu\nu} q_{\nu}$

F_1 e F_2 factores de forma

$$\frac{d\sigma}{d\Omega} \Big|_{\text{lab}} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left(\frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

$$\tau = -q^2/4M^2$$

Transf. de Fourier de G_E e $G_M \Rightarrow$ dist. de carga e momento magnético do protão

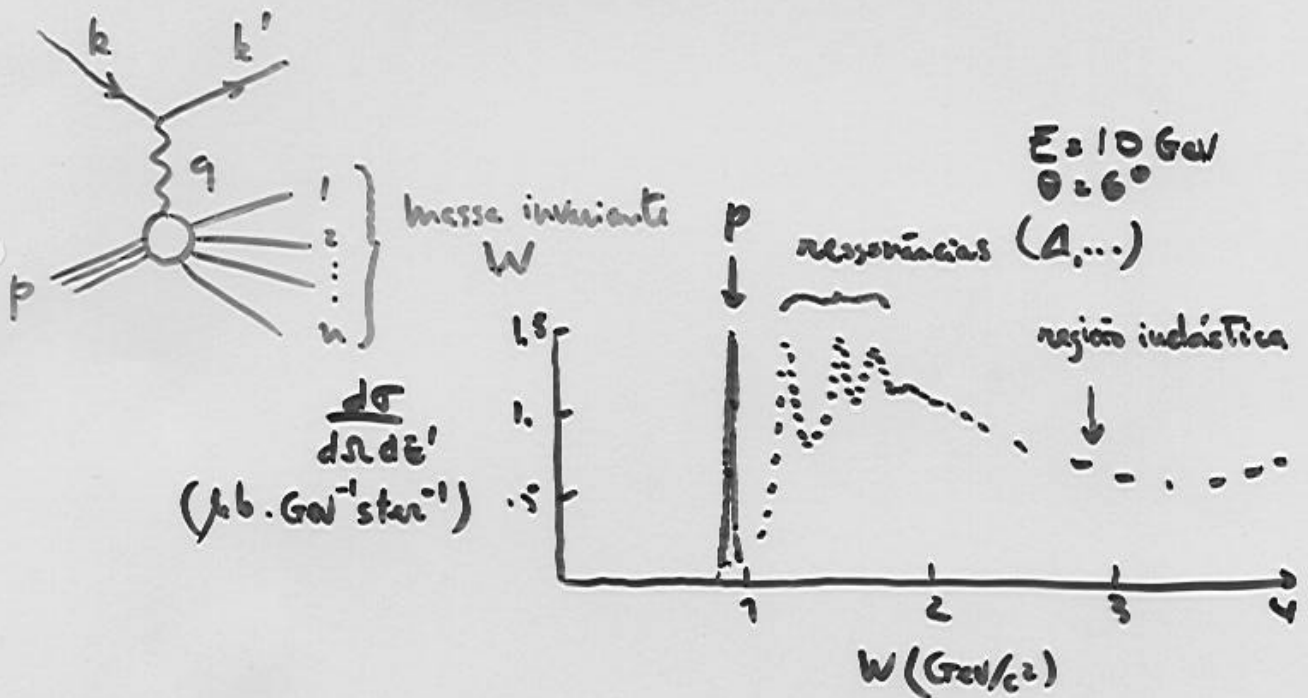
$$\langle r^2 \rangle = (-81 \text{ fm})^2$$

$$G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2 \quad ; \quad G_M \equiv F_1 + \kappa F_2$$

3. Difusão inelástica electrão - protão

Aumentar a resolução \equiv aumentar q^2

Grandes $q^2 \Rightarrow$ grande probabilidade de difusão inelástica



Forma geral de secc² eficaz:

$$d\sigma \sim L_{\mu\nu}^e W^{\mu\nu}$$

\uparrow tensor leptónico \leftarrow tensor hadrónico

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

duas variáveis independentes no vértice hadrónico:

$$q^2, \quad \gamma \equiv \frac{p \cdot q}{M} = E - E'$$

$$W^2 = M^2 + 2M\gamma + q^2$$

Difusão inelástica electrão-núcleo

Invariantes

$$\nu = \frac{P \cdot q}{M} \equiv E - E'$$

$$x = \frac{Q^2}{2 P \cdot q} = \frac{Q^2}{2 M \nu} \quad (0 \leq x \leq 1)$$

$$y = \frac{P \cdot q}{P \cdot k} = \frac{\nu}{E} \quad (0 \leq y \leq 1)$$

$$W^2 = (P + q)^2 = M^2 + 2 M \nu - Q^2$$

$$\frac{d^2 \sigma}{d\Omega dE'} = \frac{1}{2} \frac{\alpha^2}{M q^4} \frac{E'}{E} L_{\alpha\beta}(e, e') W_{e.m.}^{\alpha\beta}(N)$$

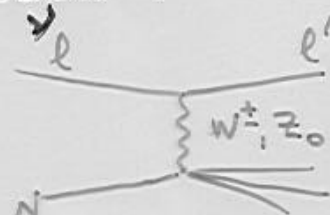
$W_{e.m.}^{\alpha\beta}(N)$ é dado em termos de duas funções: $\begin{cases} W_1^{eN}(\nu, q^2) \\ W_2^{eN}(\nu, q^2) \end{cases}$

$$\frac{d^2 \sigma}{d\Omega dE'} = \frac{\alpha^2}{4 E^2 \sin^4 \theta/2} \left[2 W_1^{eN}(\nu, q^2) \sin^2 \theta/2 + W_2^{eN}(\nu, q^2) \cos^2 \theta/2 \right]$$

Difusão inelástica neutrino-núcleo :

$$\frac{d^2 \sigma^{\nu, \bar{\nu}}}{d\Omega dE'} = \frac{G^2}{2\pi^2} \left(\frac{M^2}{M^2 + Q^2} \right)^2 E'^2 \times$$

$$\left[2 W_1^{\nu, \bar{\nu}} \sin^2 \theta/2 + W_2^{\nu, \bar{\nu}} \cos^2 \theta/2 + W_3^{\nu, \bar{\nu}} \frac{E + E'}{M} \sin^2 \theta/2 \right]$$



int e.m \longrightarrow int fraca

$$q^2 \longrightarrow q^2 - M^2$$

$$e^2 \longrightarrow \frac{G M^2}{\sqrt{2}}$$

W_3

$$M = \begin{cases} M_W & CC \\ M_Z & CN \end{cases}$$

Resumo de seções eficazes:

$e\mu \rightarrow e\mu$:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\kappa^2 E'^2}{q^4} \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2m}\right)$$

$e\mu \rightarrow ep$:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\kappa^2 E'^2}{q^4} \left(\frac{G_E^2 + 2G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2M}\right)$$

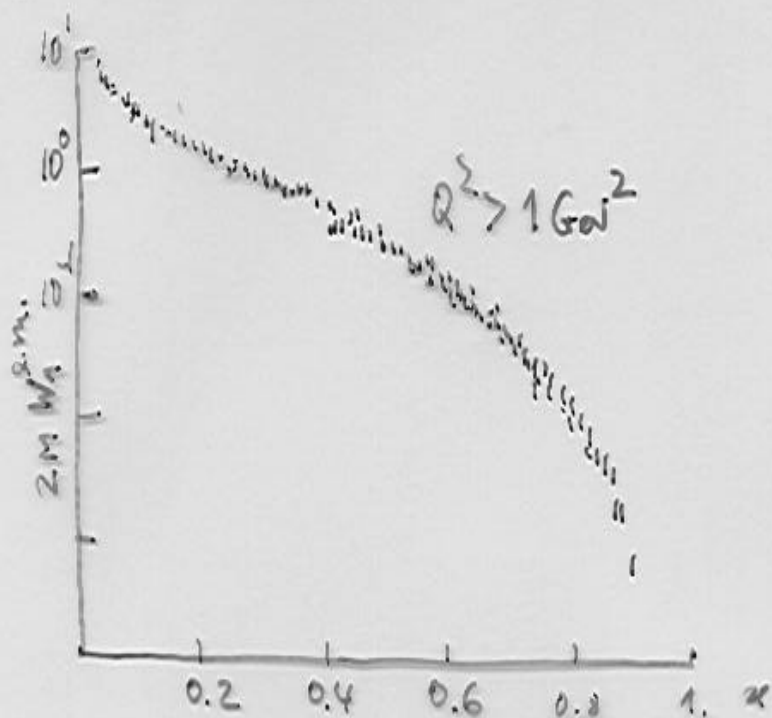
$ep \rightarrow ex$:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\kappa^2 E'^2}{q^4} \left(W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2 W_1(\nu, q^2) \sin^2 \frac{\theta}{2} \right)$$

W_1, W_2 função de estrutura

Resultados experimentais

3



Difusão electrón-núcleo:

- (ν, Q^2) (x, Q^2)
- $W_{1,2}$ independentes de Q^2

⇒ difusão sobre objectos pontuais!

Limite de Bjorken: $Q^2 \rightarrow \infty, \nu \rightarrow \infty, x = \frac{Q^2}{2m_N \nu}$ fixo

$$\lim_{Bj} m_N W_1(\nu, Q^2) = F_1(x)$$

$$\lim_{Bj} \nu W_2(\nu, Q^2) = F_2(x)$$

$$\lim_{Bj} \nu W_3(\nu, Q^2) = -F_3(x)$$

Relação entre α e ξ :

$$\alpha = \frac{Q^2}{2\tilde{P}\tilde{q}}$$



No referencial onde $\tilde{q} = (Q, 0)$:

$$Q = 2\xi P$$

$$\tilde{P} = (P, P)$$

$$\tilde{P}\tilde{q} = PQ$$

onde:

$$\alpha = \frac{Q^2}{2PQ} = \frac{Q}{2P} = \frac{2\xi P}{2P} = \xi$$

$$\alpha = \xi$$

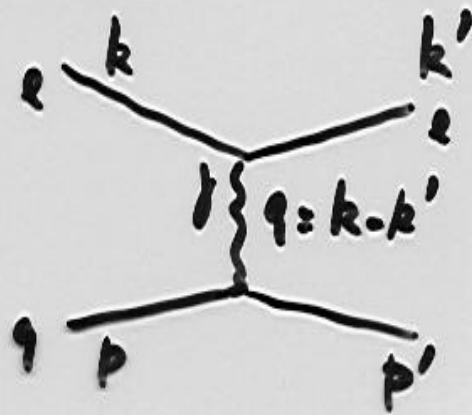
Difusão elástica electrão - quark

Cinemática:

Referencial do laboratório:

$$p = (m_q, 0), \quad k = (E, \vec{k}), \quad k' = (E', \vec{k}')$$

$$q = k - k' \quad (\epsilon - \epsilon')^2 - (\vec{k} - \vec{k}')^2 = q^2$$



$$\nu = \frac{q \cdot p}{m_q} = E - E'$$

$$q^2 \approx -4EE' \sin^2 \theta/2$$

$$Q^2 = -q^2$$

Elemento da matriz: $M \sim \underbrace{[\bar{u}_e(k') \gamma_\mu u_e(k)]}_{\text{corrente do electrão}} \frac{1}{q^2} \underbrace{[\bar{u}_q(p') \gamma^\mu u_q(p)]}_{\text{corrente do quark}}$
 propagador

$$\text{Secção eficaz: } \frac{d^2\sigma}{d\Omega dE'} = \frac{1}{2} \frac{\alpha^2 e_q^2}{2m_q q^4} \frac{E'}{E} L_{\alpha\beta}(e, e) W^{\alpha\beta}(q, q)$$

No laboratório:

$$\boxed{\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 e_q^2}{4E^2 \sin^4 \theta/2} \left(\cos^2 \theta/2 + \frac{Q^2}{2m_q^2} \sin^2 \theta/2 \right) \delta\left(\nu - \frac{Q^2}{2m_q}\right)}$$