

# A termodinâmica do Universo primitivo

Universo como um gás de partículas

$$a \ll \lambda \Rightarrow \rho_{\text{rad}} \gg \rho_{\text{m}}$$

em equilíbrio térmico!

$$\frac{\Gamma}{H} = \frac{n \bar{\sigma} v}{H} \gg 1$$

já que

$$\left. \begin{array}{l} \Gamma \propto n \propto \rho \\ H^2 \propto \rho \end{array} \right\} \frac{\Gamma}{H} \propto \rho^{-1/2} \propto a^{-2}$$

em expansão adiabática

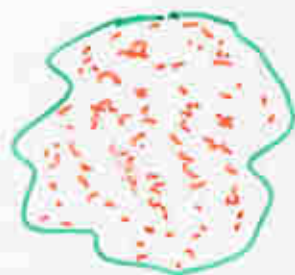
- Não há trocas de calor com o exterior...

- A variação de entropia devida a processos irreversíveis é pequena face à entropia total!

$$PV^\gamma = \text{cte} ; TV^{\gamma-1} = \text{cte}$$

$$T \propto a^{-1} \quad \text{rad.}$$

$$a^{-2} \quad \text{só mat.}$$



$\Gamma$  - taxa interação  
 $H$  - taxa expansão

[Peacock 277]



$$\frac{S}{M} \approx 10^{22} (h^2 R)^{-1} \text{ J K}^{-1} \text{ kg}^{-1}$$

$$\begin{aligned} \gamma &= \frac{c_p}{c_v} \\ &= 4/3 \quad \text{rad.} \\ &= 5/3 \quad \text{mat.} \end{aligned}$$

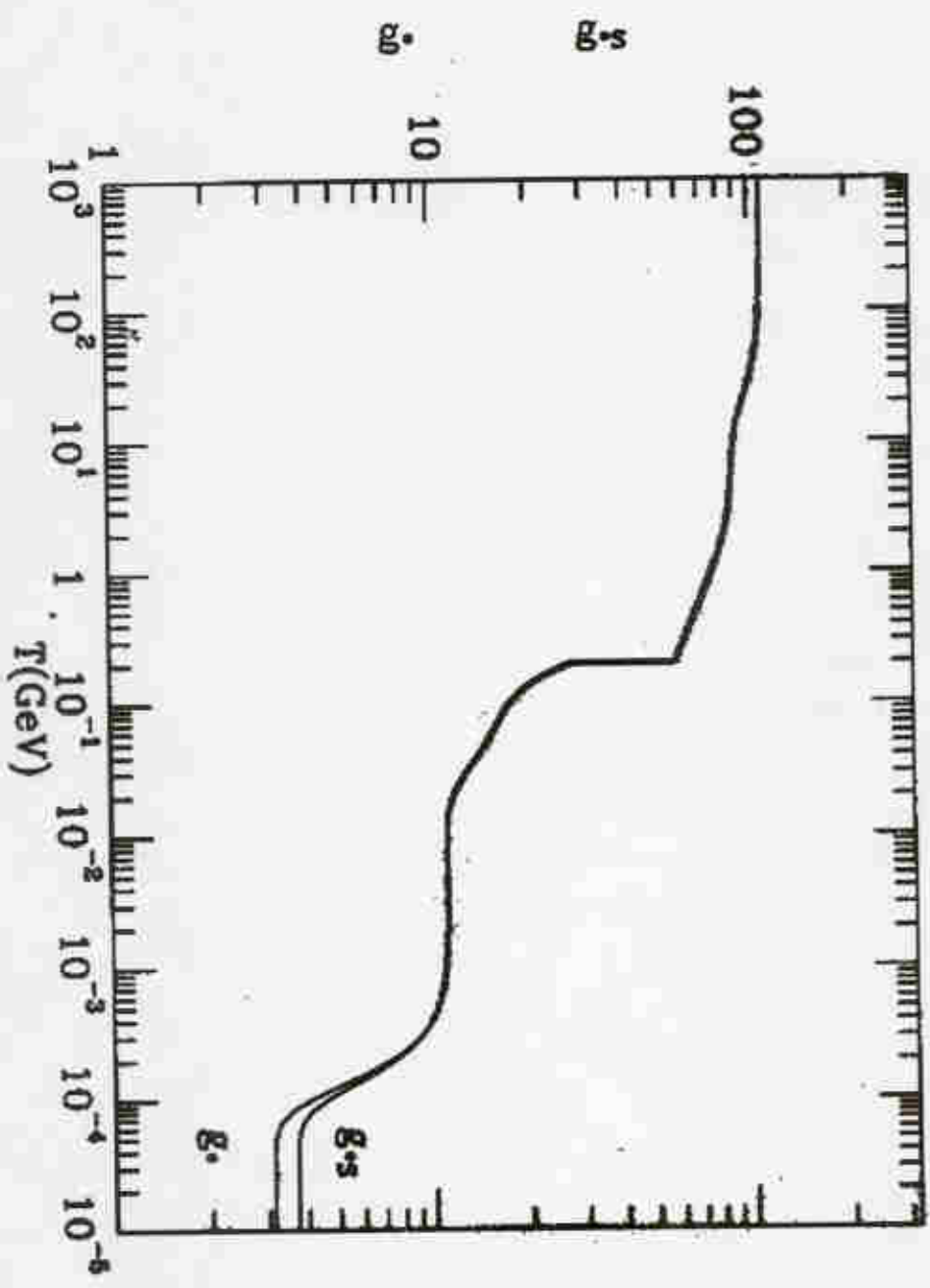
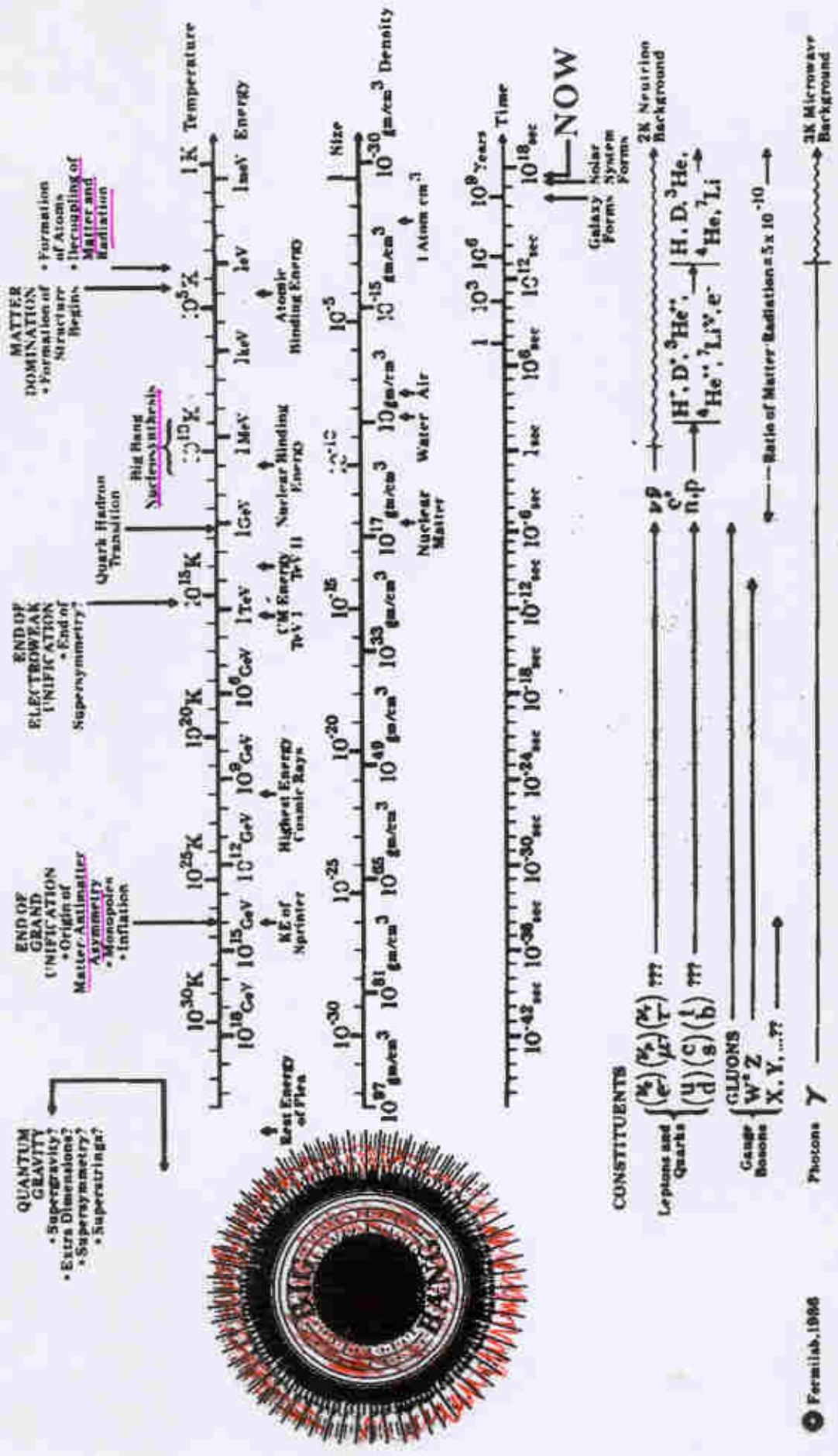


Fig. 3.5: The evolution of  $g_s(T)$  as a function of temperature in the  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  theory.

Kolb and Turner  
Early Universe



Temperatura e tempo

$$T \propto a^{-1}$$

$$a \propto t^{1/2}$$

Radiação

$$t = \left( \frac{1 \text{ MeV}}{T} \right)^2 \text{ sec}$$

Eq. de Friedmann

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_{\text{rad}}$$

$$H^2 = \frac{8\pi G}{3} \frac{\pi^2}{30} g_{\text{eff}} (k_B T)^4$$

$$\hbar = c = 1$$

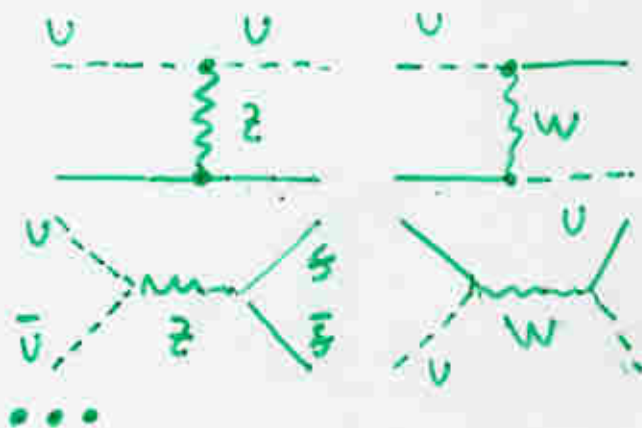
$$H = 1.66 \sqrt{g_{\text{eff}}} \frac{T^2}{M_{\text{Planck}}}$$

# Desacoplamento dos neutrinos

Interações fracas!

$$(\sqrt{s} \ll m_W)$$

$$G \sim \frac{\alpha^2 s}{m_W^4}$$



Taxa de interação

$$\Gamma \propto n G N$$

$$\Gamma \propto \frac{\alpha^2 T^5}{m_W^4}$$

$$n \propto T^3$$
$$\sqrt{s} \sim k_B T$$

Taxa de expansão

$$H = \frac{\dot{a}}{a} \propto \frac{T^2}{m_{\text{pl}}^2}$$

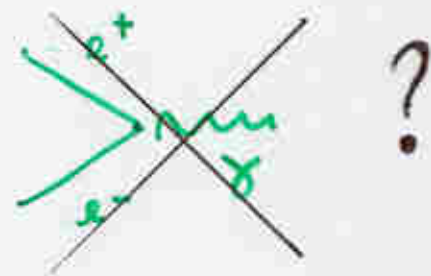
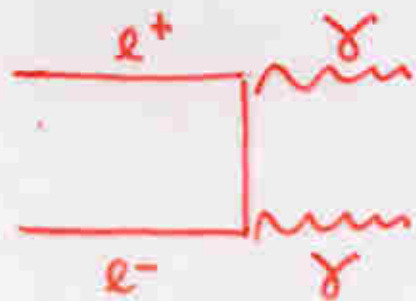
Temperatura de desacoplamento

$$\frac{\Gamma}{H} \sim 1 \Rightarrow T_D \sim \left( \frac{m_W^4}{\alpha^2 m_{\text{pl}}^2} \right)^{1/3} \sim 4 \text{ MeV}$$

Depois do desacoplamento os neutrinos estão "entregues" a si próprios!

$$T_\nu(t) = T_b \frac{a_b}{a(t)} \sim a^{-1}$$

Aniquilação  $e^+e^-$



$$e^+ e^- \rightarrow \gamma \gamma$$

$$\gamma \gamma \rightarrow e^+ e^-$$

"Reaquecimento" dos  $\gamma$ !  
 [conservação da entropia]

$$\begin{aligned} \lambda &\propto \frac{p + p}{T} \\ &\propto \frac{2\pi^2}{45} g_{\text{ef}} T^3 \end{aligned}$$

Threshold  
 $K_B T \sim 1 \text{ MeV}!$

Bergström  
 pag 138

densidade entropia

$\propto M!$

$$g_{\text{ef}} T_{\gamma R}^3 = g_{\text{ef}}^* T_{\nu}^3$$

antes:  $e^+ e^- \gamma$   
 $g_{\text{ef}} = 2 \times 2 \times \frac{7}{8} + 2$   
 depois:  $\gamma$   
 $g_{\text{ef}}^* = 2!$

$$\frac{T_{\gamma}}{T_{\nu}} = \frac{g_{\text{ef}}}{g_{\text{ef}}^*} \approx \left(\frac{11}{4}\right)^{1/3}$$

A temperatura dos neutrinos é assim  
 menor que a dos fótons!! [ $T < T_{\text{mís}}$ ]

$$T_{\gamma} / T_{\nu} \sim \left(\frac{11}{4}\right)^{1/3} \sim 1.4$$

# A formação do Deutério

Peacock  
285, 295

neutrões e prótons livres? ou ligados?

$$\frac{n_D}{n_p n_n} = \frac{3}{4} \frac{(2\pi\hbar)^3 e^{-\chi/kT}}{(2\pi kT m_p m_n / m_D)^{3/2}}$$

$$\chi = m_D - m_p - m_n$$

Energia de ligação!!!

transição abrupta

$$T_D \sim 0.1 \text{ MeV}$$

$$m_D = m_n$$

$$\frac{n_n}{n_p} \sim 0.163 (-n_B \hbar^2)^{0.04} \left(\frac{N_U}{3}\right)^{0.2}$$

Expansão ( $N_U$ )

A abundância de Hélio

$$\chi_{\text{Hélio}} > \chi_{\text{Deutério}}$$

$$\chi_{\text{Hélio}} \sim 7.1 \text{ MeV}$$

$$\chi_{\text{Deutério}} \sim 2.1 \text{ MeV}$$

$$T_{\text{Hélio}} > T_{\text{Deutério}}$$

$$T_{\text{Hélio}} \sim 0.3 \text{ MeV}$$

mas o Hélio forma-se a partir do Deutério!  
A maioria do Deutério transforma-se em Hélio

Fracção de Hélio

$$Y = 1 - \frac{m_p - m_n}{m_p + m_n}$$

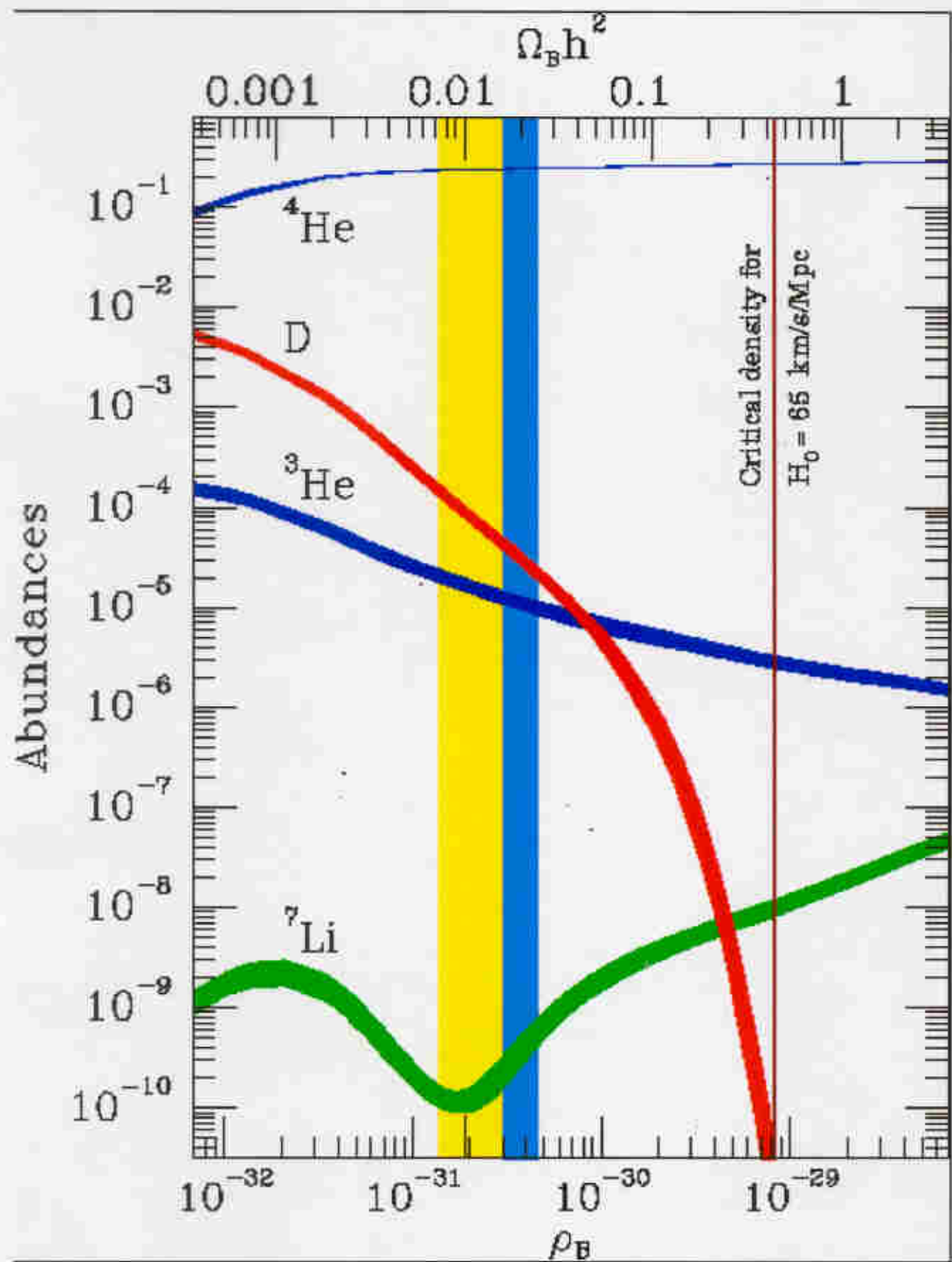
$$m_H = m_p - m_n$$

$$m_H + m_{H_2} \sim 1$$

$$\sim 0.28$$

$$\sim 0.24$$

Com correcções  
para pouco alto?



■ - Consistency interval

■ - tytler - determination of baryonic density

widths =  $2 \cdot \sigma$



Recombinação

[ O Universo passa a ser transparente quando se formam os átomos neutros ]

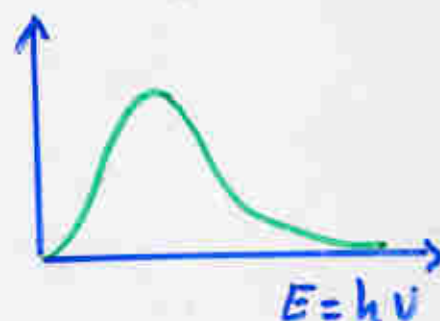
Átomo de Bohr

$$E_n = - \frac{13.6 \text{ eV}}{n^2}$$



Radiação do corpo negro

$$\lambda_{\text{max}} = \frac{b}{T}$$



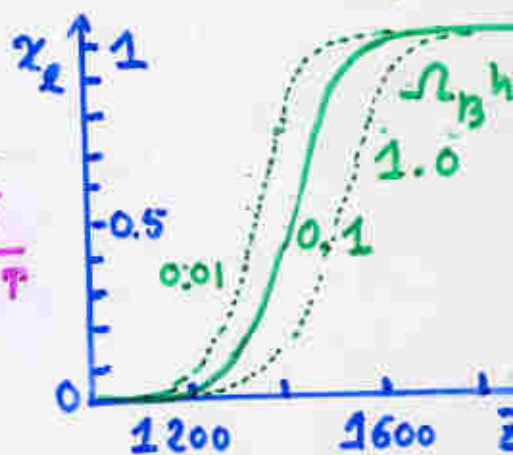
mas cauda importante

$$\eta_B = \frac{\eta_B}{\eta_\gamma} \approx 2.7 \cdot 10^{-8} \Omega_B h^2$$

Peacock leg 285

Equação de Saha (Eq. térmico)

$$\frac{1 - X_e}{X_e^2} = \frac{4 \sqrt{2} \zeta(3)}{\sqrt{\pi}} \eta_B \left( \frac{T}{m_e} \right)^{2/3} e^{-\frac{z}{kT}}$$



$$T_{\text{rec}} \approx 0.3 \text{ eV}$$

$$T = 2.73 (1+z)$$

Afasta muito do Equilíbrio

Ionização residual

$$X_e \gtrsim 10^{-4}$$

Desacoplamento  $\gamma$ /matéria

$$z \sim 1065$$

[ última superfície de difusão

$$z \gg a(t)$$

## Exercícios Revisão 29/3

- 1) Determine a temperatura ( $^{\circ}\text{K}$ ) e a densidade de energia média dos  $\gamma$  e  $\nu$  no universo logo logo após os eventos de Hidrogeno.
- 2) Compare as densidades de energia ~~energias~~ ~~potenciais~~ das galáxias e de radiação cósmica de fundo no Universo.
- 3) Mostre que o espectro de fótons de radiação cósmológica de fundo ~~é~~ <sup>permanece</sup> ~~é~~, após o desacoplamento, uma distribuição de corpo negro.
- 4) Determine a temperatura e a densidade de energia de uma possível componente "gravitão" de radiação cósmológica de fundo. Considere que a seção eficaz dos gravitons é de ordem de  $\frac{8}{3} \cdot \frac{4}{k_{\text{Planck}}^2}$ .
- 5) Problema 13.13

# Shu - Pb. 13.13: A Luminosidade de Eddington

\* Buraco negro supermaciço } após ionização da matéria atraída,  
\* Binárias próximas } equilíbrio entre pressão gravítica e radiativa

320

QUIET AND ACTIVE GALAXIES

arise for cosmological reasons, and reject Arp's findings as chance superpositions of a nearby galaxy and a distant quasar. Many people have argued about the statistical significance of each camp's results, but quasar statistics are notorious for being of low moral character: one can do anything with them. Schmidt, Weedman, and others have presented a good case that we have no compelling reason to treat quasars separately from a general class of active galactic nuclei, for which the cosmological interpretation of the redshift is not in doubt.

## Supermassive Black-Hole Models for Active Galactic Nuclei

That an object can change its luminosity by more than twice within a short time argues strongly that the basic machine in the nucleus of a Seyfert, or a radio galaxy, or a quasar, is a single body. The most promising theoretical candidate is an accreting supermassive black hole. This concept, originally by Edwin Salpeter and Donald Lynden-Bell, has been applied on a smaller scale to explain the emission from the binary X-ray source, Cygnus X-1 (Chapter 10). To play a similar role in active galactic nuclei, the black hole has to be supermassive just to be able to overwhelm the enormous radiation pressure which tries to push the accreting matter back out (Problem 13.13). For a luminosity of  $10^{47}$  erg/sec, the mass of the black hole has to be larger than about  $10^9 M_\odot$ .

**Problem 13.13.** The quantity  $r_e = e^2/m_e c^2$  is called the classical radius of the electron, not because the electron actually has finite size, but because a free electron interacting with photons scatters them as if it had a radius of  $r_e$ . In particular, if the photons have much lower energies than the rest energy of the electron, the scattering cross section of free electrons is  $8\pi r_e^2/3$ . Idealize the gas near a quasar to be completely ionized hydrogen which is optically thin to the radiation emerging from the central source of luminosity  $L$ . Show that each free electron intercepts per unit time the energy  $(8\pi r_e^2/3)(L/4\pi r^2)$ , and scatters this energy in all directions. Assume that only the interception process leads to a net transfer of momentum, and show that, since the momentum carried by a photon is  $1/c$  times its energy, the time-rate of transfer of momentum per free electron is given by  $2r_e^2 L/3cr^2$ . This outward push of the electrons by the radiation pressure acts also indirectly on the protons (the nuclei of the ionized hydrogen atoms), because strong electrostatic forces tend to make the electron gas and proton gas move

together. Countering the outward force  $2r_e^2 L/3cr^2$  acting on each electron-proton pair is the inward pull of gravity  $GM(m_p + m_e)/r^2$ , where  $M$  is the mass of the central supermassive black hole (we assume  $r$  is far from the event horizon of the black hole). Argue that for accretion to occur, the pull of gravity must be larger, and that this condition requires

$$M \geq \frac{2r_e^2 L}{3Gm_H c}$$

where  $m_H = m_p + m_e$  is the mass of the hydrogen atom. Compute the right-hand side when  $L = 10^{47}$  erg/sec. For a given  $M$ , the value of  $L$  which balances this equation is called the Eddington luminosity  $L_E$ . Compute  $L_E = 3GMm_H c/2r_e^2$  for a stellar mass  $M$ . How does this compare with the maximum X-ray luminosity  $\sim 10^{38}$  erg/sec seen in binary X-ray sources? (Consult Chapter 10.) Draw appropriate astronomical conclusions.

To produce  $L = 10^{47}$  erg/sec (via a swirling accretion disk, say), this supermassive black hole has to swallow more than  $10M_\odot$  per year (Problem 13.14). It may acquire this matter by breaking up whole stars by tidal forces, or by ingesting interstellar gas left over from star formation or subsequently ejected from evolving stars. If the density of stars is sufficiently high near the black hole, direct collisions of stars may contribute to the diet of the black hole. Rees has emphasized that there are many possible ways in which such a black hole might have first formed and by which it might subsequently acquire more material to swallow.

**Problem 13.14.** For every mass  $m$  which a black hole swallows (via an accretion disk, say), an amount of energy  $\epsilon mc^2$  is liberated, where  $\epsilon$  is the efficiency of the process. The maximum possible value for  $\epsilon$  is unity, but a value equal to 10 percent might be more realistic. At what rate  $\dot{M}$  would a supermassive black hole have to swallow mass to produce  $L = 10^{47}$  erg/sec if  $\epsilon = 0.1$ ? Convert your answer to  $M_\odot/\text{yr}$ .

Some people object to the idea of supermassive black holes because they feel it is a very weird concept. Yet, in one important sense, supermassive black holes are much less weird than stellar-mass black holes. Consider the density of the pre-black-hole object which is doomed to collapse to singularity. In order of magnitude, this density must equal the mass  $M$  divided by the volume  $V$ , for