O modelo "Padrão" da Cosmologia

FLRW - Friedmann, Lemaitre
Robertson e Walker

Espaço tempo homogeneo e isotrópico

$$ds^2 = dt^2 - a(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right)$$

Resolvendo agora as Eq. de Einstein com esta métrica obtêm-se as Eq. de Friedmann:

$$\boxed{\begin{aligned} \left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} &= \frac{8\pi G}{3}\rho \\ \frac{2\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} &= -8\pi G p \end{aligned}}$$

$$\frac{\dot a}{a} = H = \text{cte Hubble}$$

que podem ser ainda expresses como:

$$\frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}(a^3)$$

conservação Energia

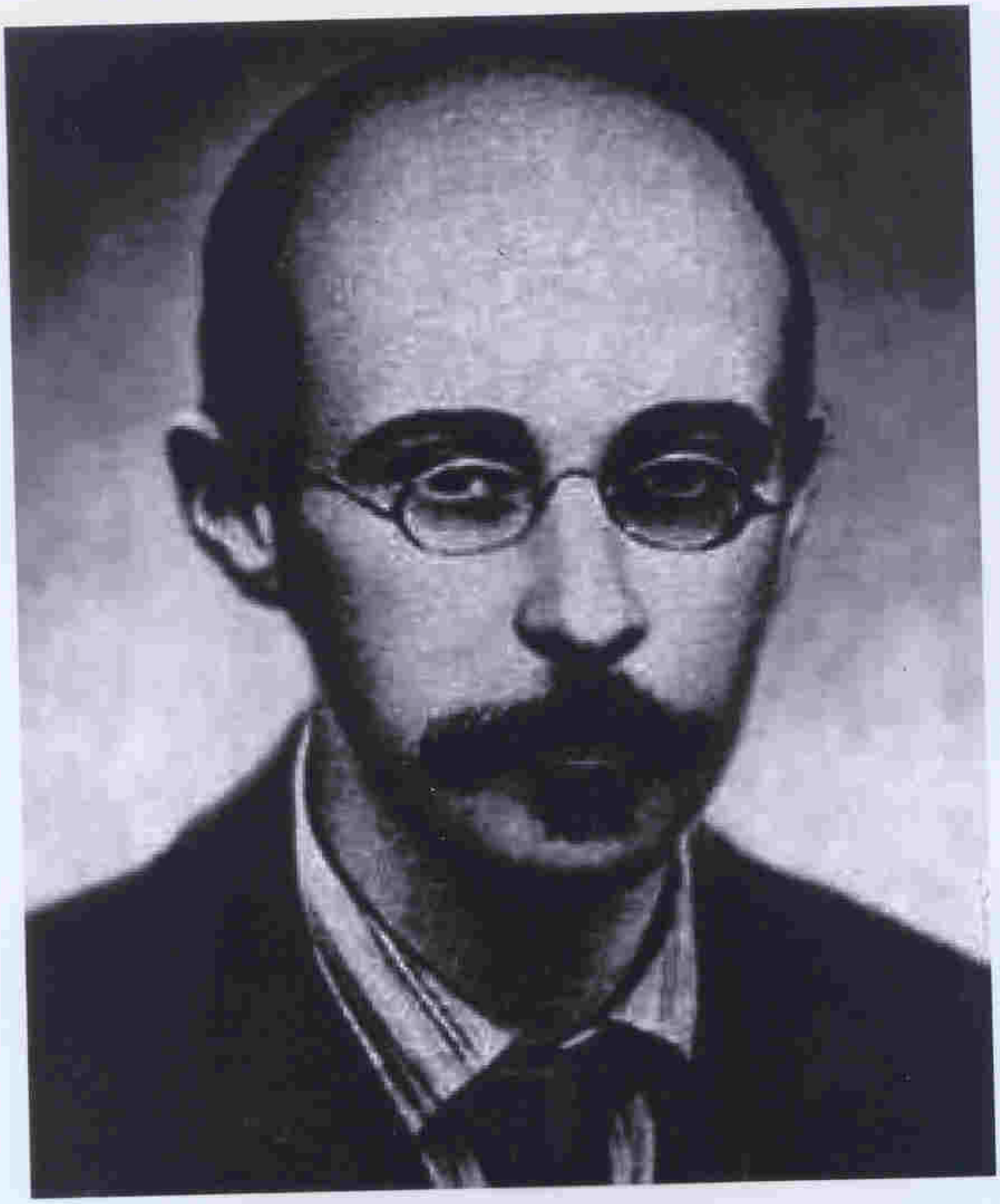
$$\frac{d}{dt}\left(\frac{a^3}{T}(\rho(T)+p(T))\right) = 0$$

conservação Entropia

$$S = \frac{V}{T}(\rho + p)$$

e ainda

$$\boxed{\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)}$$



Alexander Friedmann
1888-1925

Newton ...

- Da lei de Newton

$$m\ddot{R} = -\frac{GMm}{R^2}$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G \rho}$$

interior de uma esfera!

- Da conservação de Energia

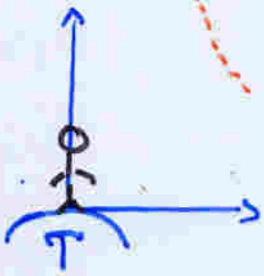
$$\frac{1}{2} m \dot{R}^2 - \frac{GMm}{R} = cte$$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 - \frac{cte}{a^2} = \frac{8\pi}{3} G \rho}$$

- Na mecânica clássica a pressão não contribui para a "massa gravitacional"
- A curvatura espacial está relacionada com −(Energia total) ou seja com a Energia de ligação do sistema

Fuga para o vermelho (redshift)

Raios de luz radial
em geodésicas $ds^2 = 0$!
$(d\varphi = d\theta = 0)$



$$c\,dt = a(t)\,\frac{dr}{\sqrt{1-kr^2}}$$

Uma primeira frente de onda emitida pela estrela (t_E, r_E) chega ao observador $(t_0, 0)$

$$\int_{t_E}^{t_0} \frac{dt}{a(t)} = \frac{1}{c}\int_0^{r_E} \frac{dr}{\sqrt{1-kr^2}}$$

Para uma segunda frente de onda $(t_E + \Delta t_E, r_E)$

$$\int_{t_E+\Delta t_E}^{t_0+\Delta t_0} \frac{dt}{a(t)} = \frac{1}{c}\int_0^{r_E} \frac{dr}{\sqrt{1-kr^2}}$$

e portanto

$$\int_{t_E}^{t_0} \frac{dt}{a(t)} = \int_{t_E+\Delta t_E}^{t_0+\Delta t_0} \frac{dt}{a(t)} \Rightarrow \frac{\Delta t_0}{\Delta t_E} \sim \frac{a(t_0)}{a(t_E)}$$

$$z = \frac{\lambda_0 - \lambda_E}{\lambda_E}$$

$$\boxed{1 + z = \frac{\lambda_0}{\lambda_E} = \frac{a(t_0)}{a(t_E)}}$$

$\neq$ Doppler!!!
Σ Dopplers infinitesimais
$z \sim v$ sse $z \ll$

Universo estático

$\dot{a} = 0$

$$\frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\rho = \frac{3k}{8\pi G a^2} \qquad k = \underline{1} \; !$$

$\ddot{a} = 0$

$$(\rho + 3p) = 0$$

$$p = -\frac{1}{3}\rho \qquad \text{Pressão negativa!}$$

genericamente (equação de estado)

$$p = \alpha\rho \qquad \alpha \simeq 0 \quad \text{matéria}$$

$$= \frac{1}{3} \quad \text{radiação}$$

É necessário introduzir um novo tipo de "entidade" - o vácuo!

$$p = -\rho \qquad \alpha = -1$$

Num Universo estático de matéria não relativista e vácuo:

$$\underline{\rho_m = 2\rho_{vácuo}}$$

A constante cosmológica

Na construção da Eq. de Einstein

$$\partial_\nu G^{\mu\nu} = \partial_\nu T^{\mu\nu} = 0$$

mas

$$\partial_\nu \left(G^{\mu\nu} + \Lambda g^{\mu\nu} \right) = 0$$

Eq. mais geral

$$\boxed{G^{\mu\nu} + \Lambda g^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu}}$$

ou ainda

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} \left[T^{\mu\nu} + T_\Lambda^{\mu} \right]$$

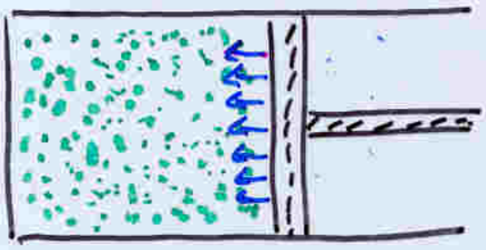
$$T_\Lambda^{\mu\nu} = \begin{bmatrix} \rho_\Lambda & & & \\ & -\rho_\Lambda & & \\ & & -\rho_\Lambda & \\ & & & -\rho_\Lambda \end{bmatrix} \qquad \rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

$$p_{vacuo} = -\rho_{vacuo}$$

$$dU = -p\, dV$$

$$dU = \rho\, dV$$

$$\boxed{\rho = cte}$$



Pressão negativa!

E ainda a constante cosmológica

Num Universo sem matéria ou radiação (só vácuo!) - de Sitter

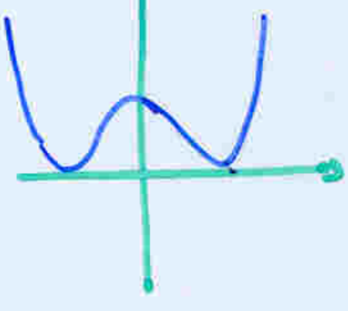
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(-2\rho_{vac}\right)$$

$$= \Lambda/3 \qquad \text{cte} > 0$$

$$\boxed{a(t) \simeq e^{\chi t}} \qquad \chi = \sqrt{\Lambda/3}$$

- A constante cosmológica desempenha o papel duma "massa" negativa, induz uma força de repulsão
- Num Universo dominado pela constante cosmológica a expansão é <u>exponencial</u>!!!

A existência de quebra de simetria em potenciais escalares (mecanismo de Higgs) pode induzir ρ_{vac} enormes!!!



$$V(\phi) \sim -\mu^2|\phi|^2 + \lambda\phi^4$$

$$\rho_{vácuo} \sim V_{min} \sim \mu^4/4\lambda$$

$$\sim (200\ \text{GeV})^4 \qquad \text{M.S.}$$

$$\sim (10^{19}\ \text{GeV})^4 \qquad \text{Planck}$$

mas experimentalmente $\rho < 10^{-46}\ \text{GeV}^4$!!!

Universos de uma só componente

- Dependência de "ρ" com "a"

$$\begin{cases} \frac{d}{dt}(\rho a^3) = -p \frac{d}{dt} a^3 \\ p = \alpha \rho \end{cases}$$

Conservação Energia

Eq. de estado

$$\boxed{\rho \propto a^{-3(1+\alpha)}}$$

- Dependência de "a" com "t"

$$\begin{cases} \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \\ a(t) \sim t^{\beta} \end{cases}$$

$$\boxed{a \sim t^{\frac{2}{3(1+\alpha)}}}$$

- Universos

	α	ρ	$a(t)$ $t \ll$
matéria	0	a^{-3}	$t^{2/3}$
radiação	$1/3$	a^{-4}	$\sqrt{t}$
"vácuo"	-1	cte	$[e^{Ht}]$
"curvatura"	$-1/3$	a^{-2}	t
???	$[-1, 0]$		

Radiação → matéria → ... vácuo !!! ($\Lambda \neq 0$)

Densidades "normalizadas" (Ω)

Eq. Friedmann

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$$

$K = 0$

$$\boxed{\rho_{crit} = \frac{3H^2}{8\pi G}} \qquad H = \frac{\dot{a}}{a}$$

Definindo densidades "normalizadas"

$$\Omega_i = \frac{\rho_i}{\rho_{crit}} \qquad \Omega_{tot} = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$$\frac{K}{H^2 a^2} = \Omega - 1 \qquad K = 0 \Leftrightarrow \Omega_{tot} = 1$$

e ainda por "analogia"

$$\Omega_K = -K/\dot{a}^2 \qquad \rho_K = \frac{-3K}{8\pi G}\frac{1}{a^2}$$

$$\boxed{\begin{gathered}\Omega_m + \Omega_{rad} + \Omega_\Lambda + \Omega_K = 1 \\ H^2 = H_0^2\left[\Omega_\Lambda + \Omega_K a^{-2} + \Omega_m a^{-3} + \Omega_{rad} a^{-4}\right]\end{gathered}}$$

Eq. de Friedmann

Universo de matéria

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Peacock

- $\Omega_m \gg \Omega_{Rad}$ e $\Omega_\Lambda = 0$

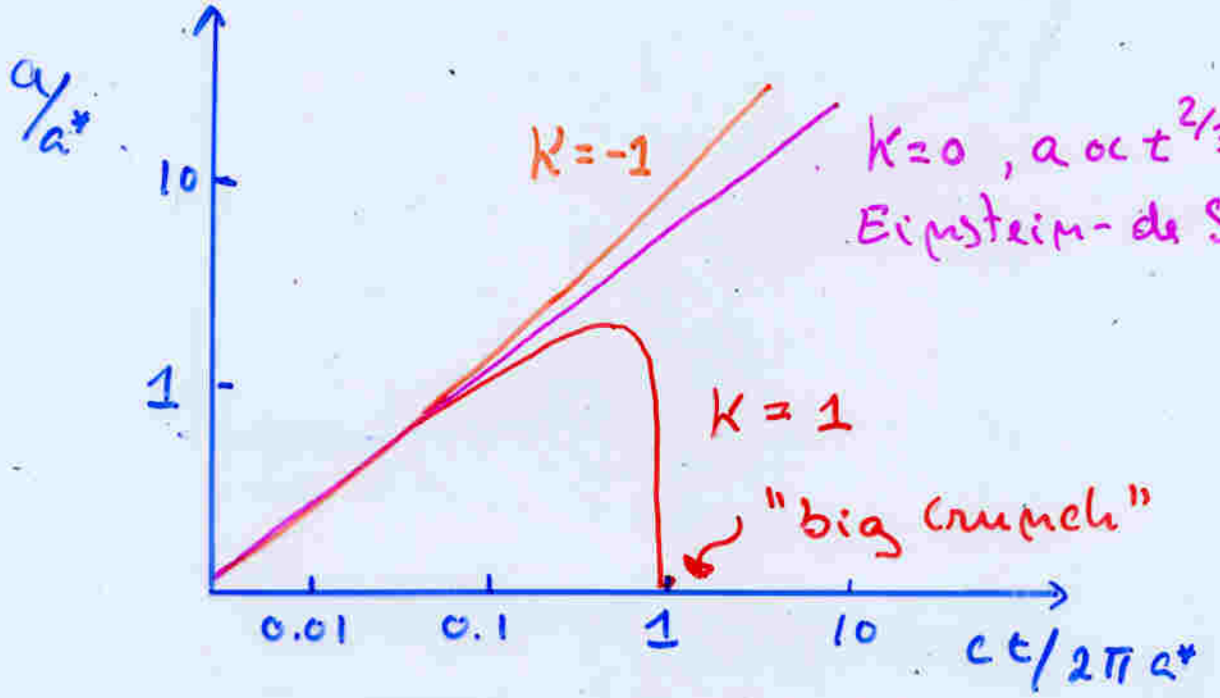
- $\Omega_m > 1 \Rightarrow k = 1$
  $\Omega_m = 1 \Rightarrow k = 0$
  $\Omega_m < 1 \Rightarrow k = -1$

$$\frac{k}{H^2 a^2} = \Omega - 1$$

A eq. de Friedmann reduz-se a:

$$\dot{a}^2(t) = \frac{8\pi G \rho_0 a_0^3}{3\, a(t)} - k$$

de solução:



Univ. Radiação versus Univ. matéria

$$\rho_{rad}(a) = \rho_{rad}(a_0)\left(\frac{a}{a_0}\right)^{-4}$$

$$\rho_{mat}(a) = \rho_{mat}(a_0)\left(\frac{a}{a_0}\right)^{-3}$$

$$\Rightarrow \quad \rho_{Rad} = \rho_{mat}$$

$$1+z = a/a_0 = \frac{\Omega_m(0)}{\Omega_{rad}(0)} \simeq 20.000$$

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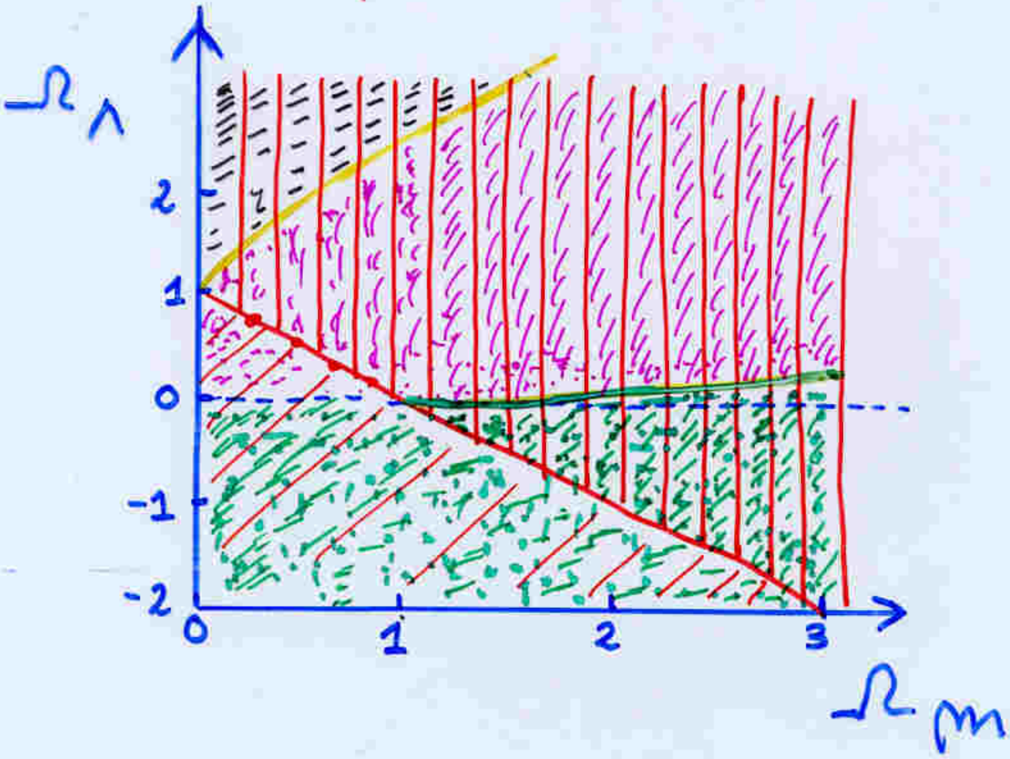
Peacock

O destino do Universo

determinado por $(\Omega_m, \Omega_\Lambda)$

[quando a grande $\Omega_{rad} \ll \Omega_m$]



/// - aberto $(\Omega_\Lambda + \Omega_K < 1 \Rightarrow K = -1)$

— - plano (" $= 1$ $K = 0$)

||| - fechado (" > 1 $K = +1$)

- Recolapsa $(\Lambda_\Lambda < 0 \vee (\Omega_m > 1 \wedge \Omega_\Lambda < \Omega_c))$

- Expande para sempre

- Não existe big-bang

- Oscilatório

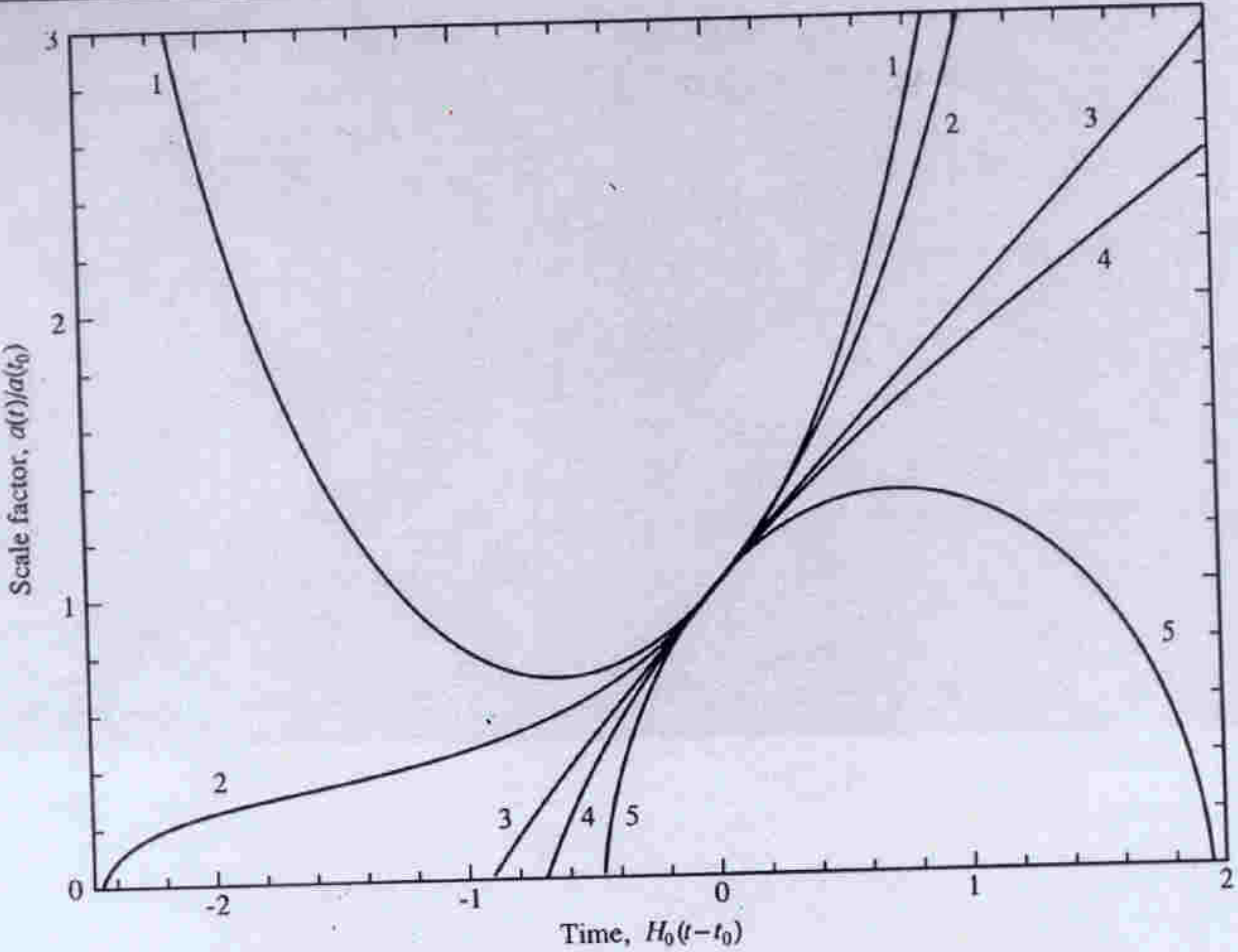
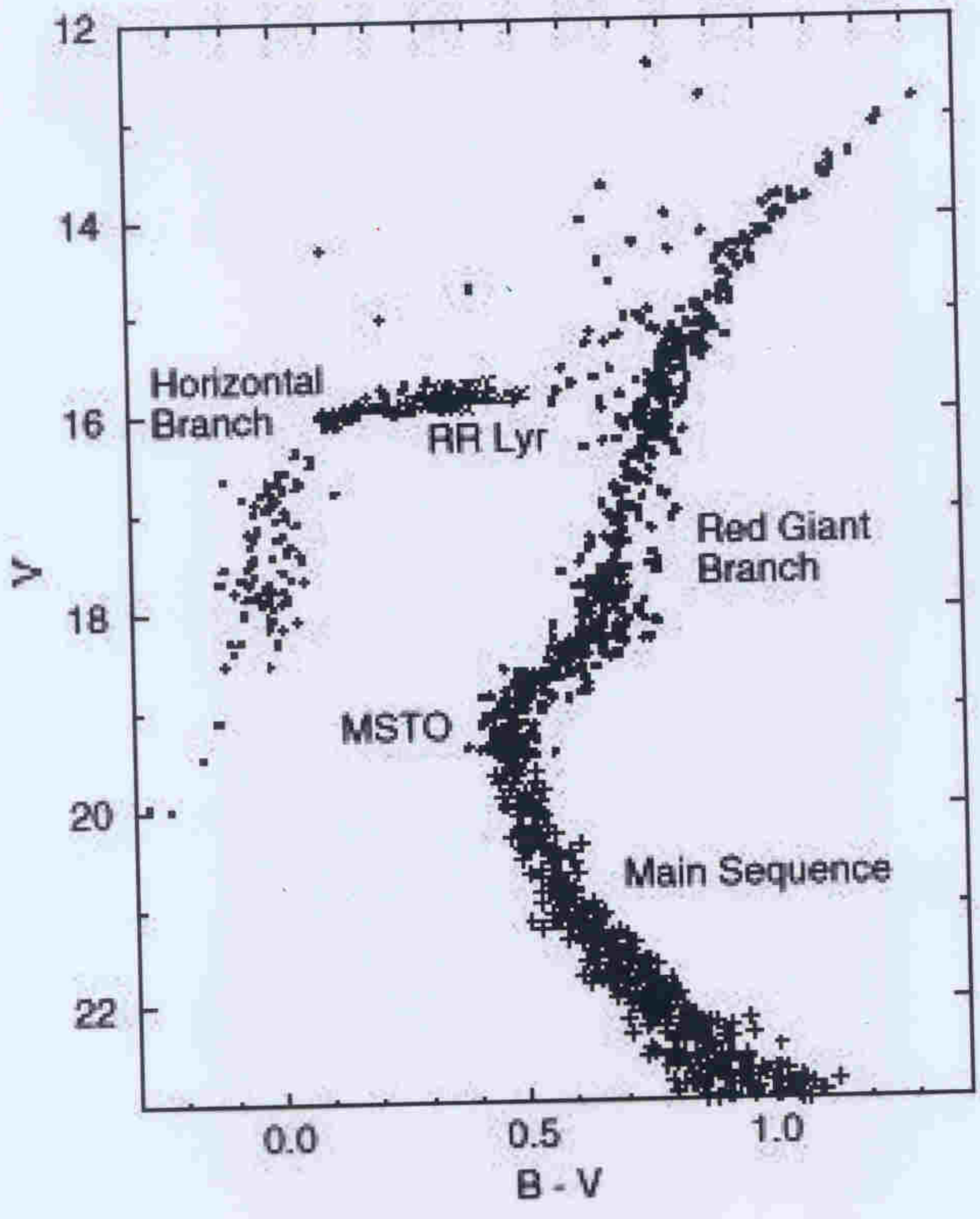


Figure 2.3 The five types of currently expanding universe, showing their history and fate plotted in scale factor vs. time. A Big Bang model has $a = 0$ in the past. The cases are (1) bounce universe: must have some $\sigma < -1/3$; (2) Eddington–Lemaître universe: has *coasting* phase, must have some $\sigma < -1/3$; (3) open Universe: $\Omega < 1$ in $\sigma > -1/3$; (4) critical (Einstein–de Sitter) universe: $\Omega = 1$ in $\sigma > -1/3$; (5) closed universe: $\Omega > 1$ in $\sigma > -1/3$. Note that model 4 has a asymptotically constant (horizontal).

$(\sigma \equiv \alpha)$


12
14
16
18
20
22
V
0.0
0.5
1.0
B - V
Horizontal
Branch
RR Lyr
Red Giant
Branch
MSTO
Main Sequence

A idade do Universo

Peacock
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$$T_{univ} = \int_0^{\infty} \frac{dt}{dz}\, dz$$

$z = 0$ presente

$z = \infty$ "big bang"

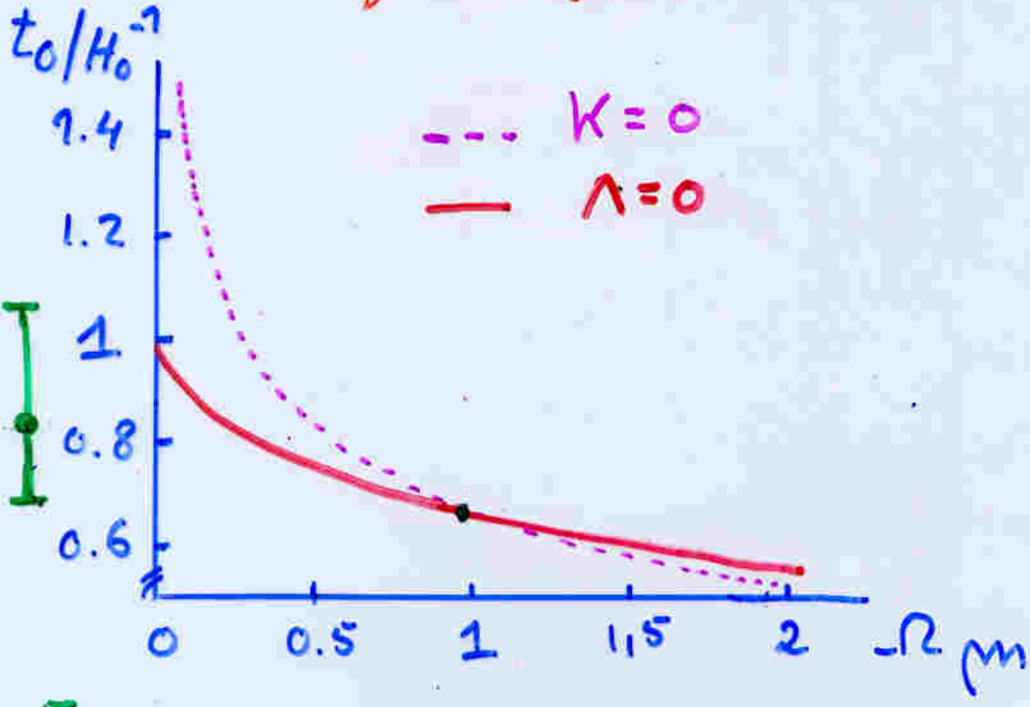
mas

$$H = \frac{d}{dt} \log\left(\frac{a(t)}{a_0}\right) = \frac{-1}{1+z}\frac{dz}{dt}$$

$$H^2 = H_0^2\left[\Omega_\Lambda + \Omega_K a^{-2} + \Omega_m a^{-3} + \Omega_{Rad} a^{-4}\right]$$

e portanto

$$T \equiv f(H_0, \Omega_i)$$



Ω_{Rad} desprezável

$K=0,\ \Lambda=0$

Einstein-de Sitter

$t_0 = \frac{2}{3} H_0^{-1}$

$\simeq 10^{10}$ anos

Evolução das estrelas

sec XIX : T_{Sol} 10^8 anos!!! Kelvin e Helmholtz

hoje : Estrelas na sequência principal

(L. Krauss) (Luminosidade versus côr)

$$T \sim 12.7^{+3}_{-2}\ 10^9 \text{ anos}$$

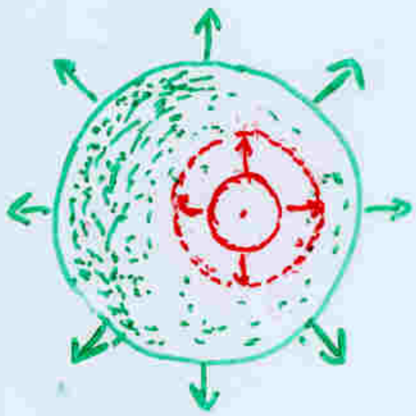
Expansão do factor de escala

$$a(t) = a(t_0)\left(1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2 + \dots\right)$$

$$H_0 = \frac{\dot{a}(t_0)}{a(t_0)}$$ cte de Hubble

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0) H_0^2}$$ parâmetro de desaceleração

lei de Hubble



$$v = \lim \frac{d_2 - d_1}{t_2 - t_1}$$

$$= \frac{\dot{a}}{a} \, a s$$

$$\boxed{v = H d}$$

Relação entre q_0 e os Ω_i

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$q_0 = \frac{1}{2}\Omega_0 + \frac{3}{2}\sum \alpha_i \Omega_i$$

$$\boxed{q_0 = \frac{1}{2}\Omega_m + \Omega_{Rad} - \Omega_\Lambda}$$

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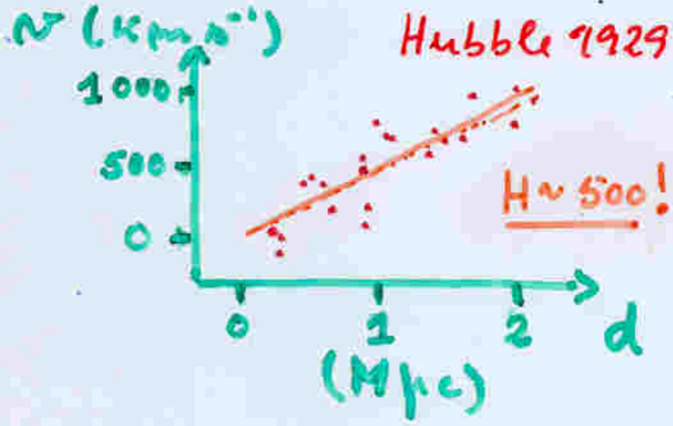
Bergström

Determinação de H_0 e q_0

idealmente

"v" do redshift z

"d" da luminosidade



mas

$$d_L = \sqrt{\frac{L}{4\pi \mathcal{L}_{ap}}}$$

conhecer o L ?
"velas padrão"

$$d_L = a(t_0)\, r_1 (1+z)$$

Redshift da luz e dilatação do tempo

$$\frac{a_0\, dr}{\sqrt{1-kr^2}} = (1+z)\, dt$$

geodésica $ds^2 = 0$

$$\frac{dr}{dt} = \frac{\sqrt{1-kr^2}}{a(t)}$$

$$a_0 \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \int_0^{z_1} \frac{dz}{H_0\left[(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda\right]^{1/2}}$$

d_L é uma função $(z, H_0, \Omega_M, \Omega_\Lambda \ldots)$

Valores actuais (L. Krauss)

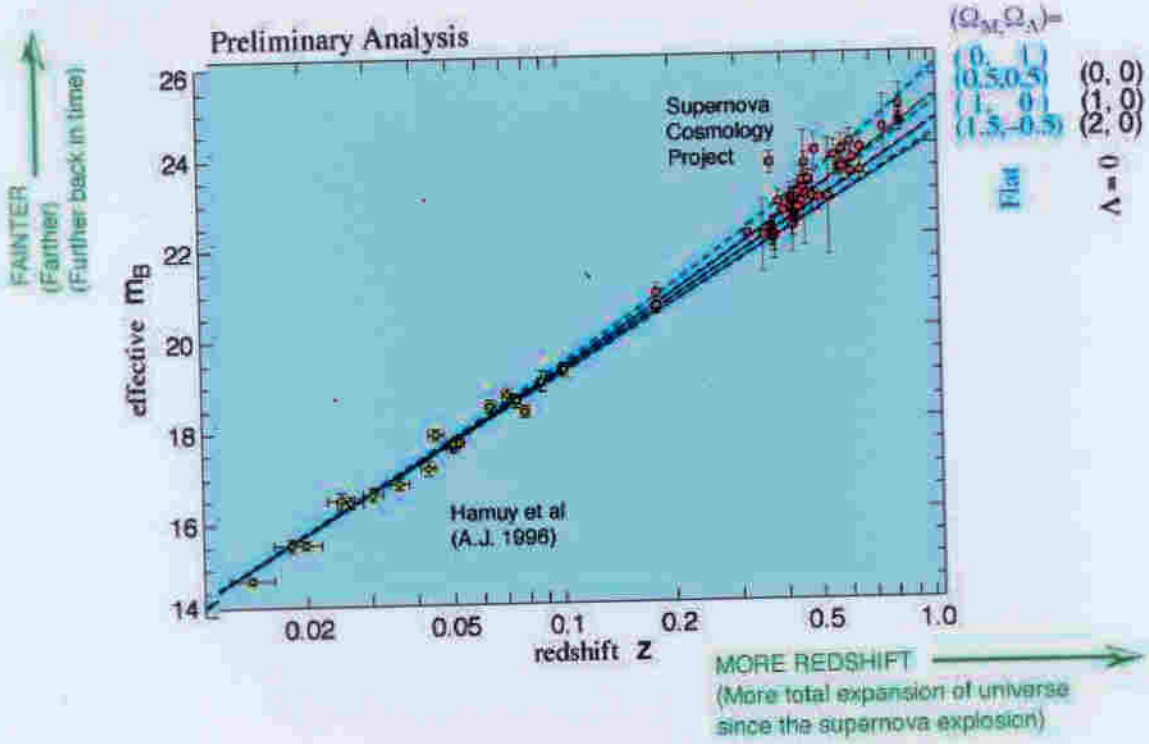
$$\boxed{H_0 \simeq 68 \pm 3 \ \mathrm{K\, s^{-1}\, Mpc^{-1}}}$$

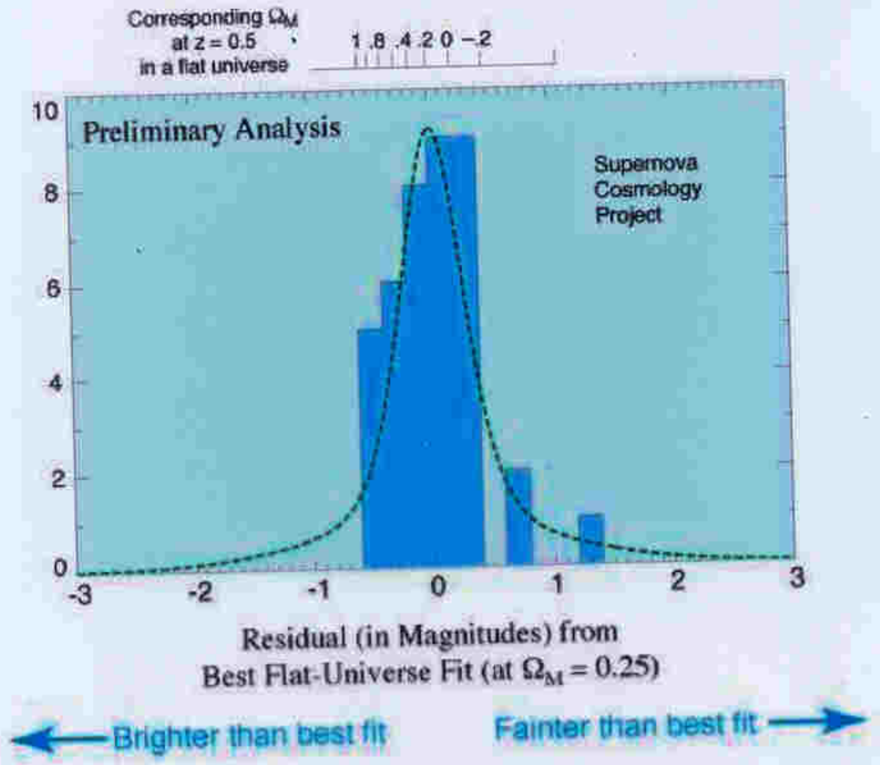
HST-KEY
S-Z efeito
Type 1a SN

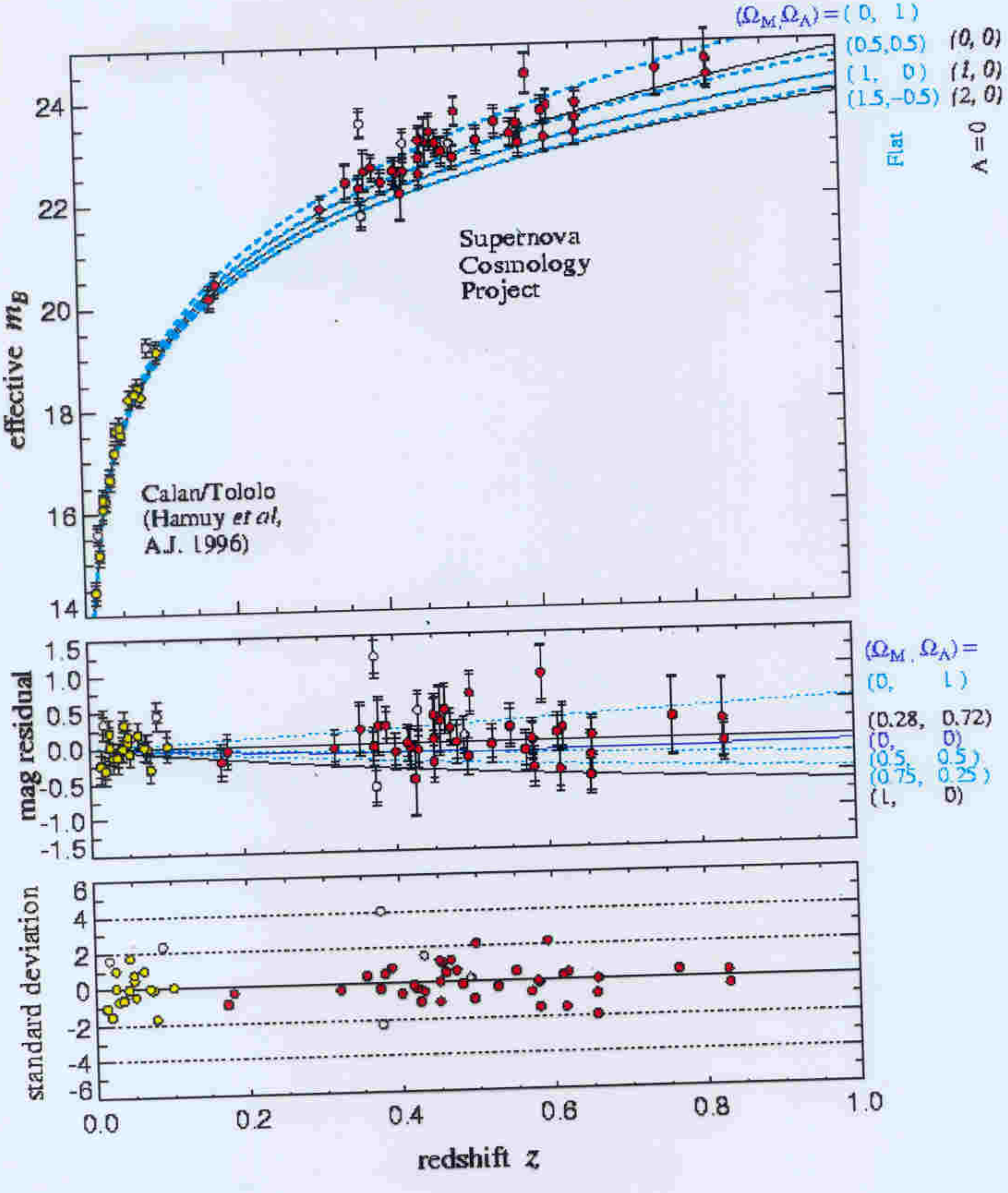
$$\boxed{q_0 < 0}$$

$$0.8\,\Omega_M - 0.6\,\Omega_\Lambda \simeq -0.2 \pm 0.1$$

Hubble Plots

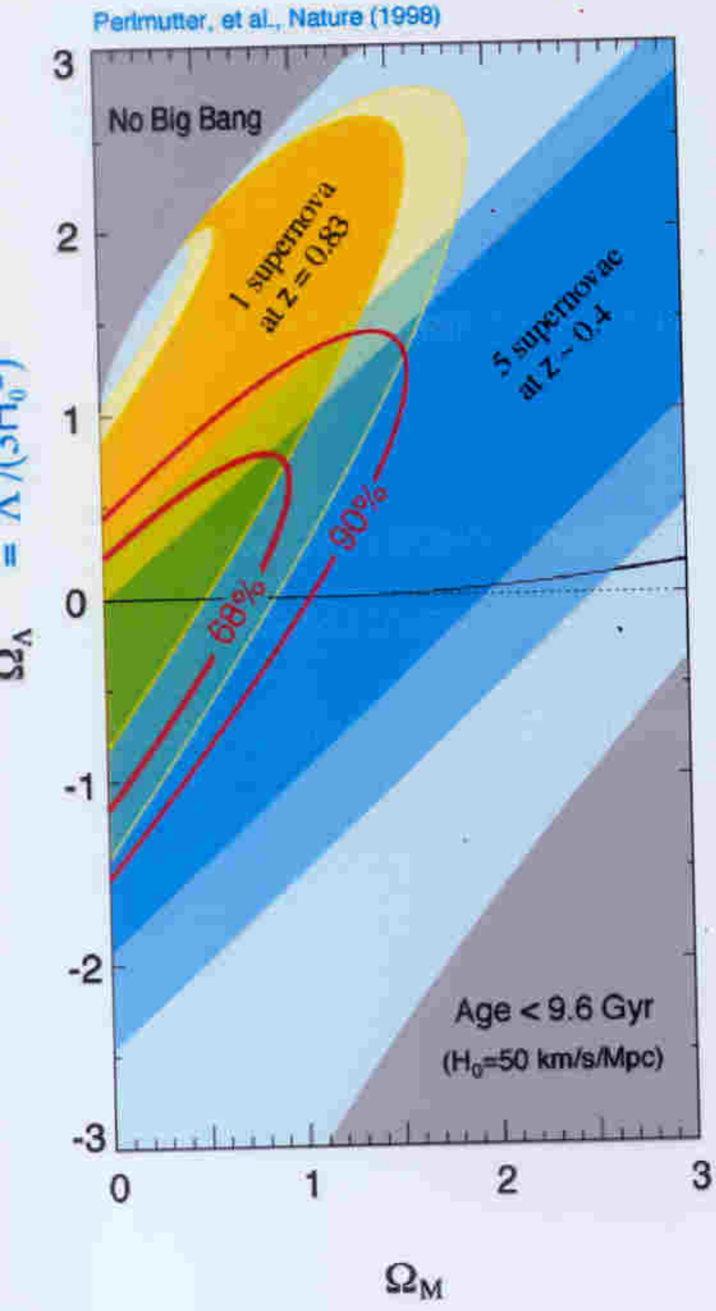




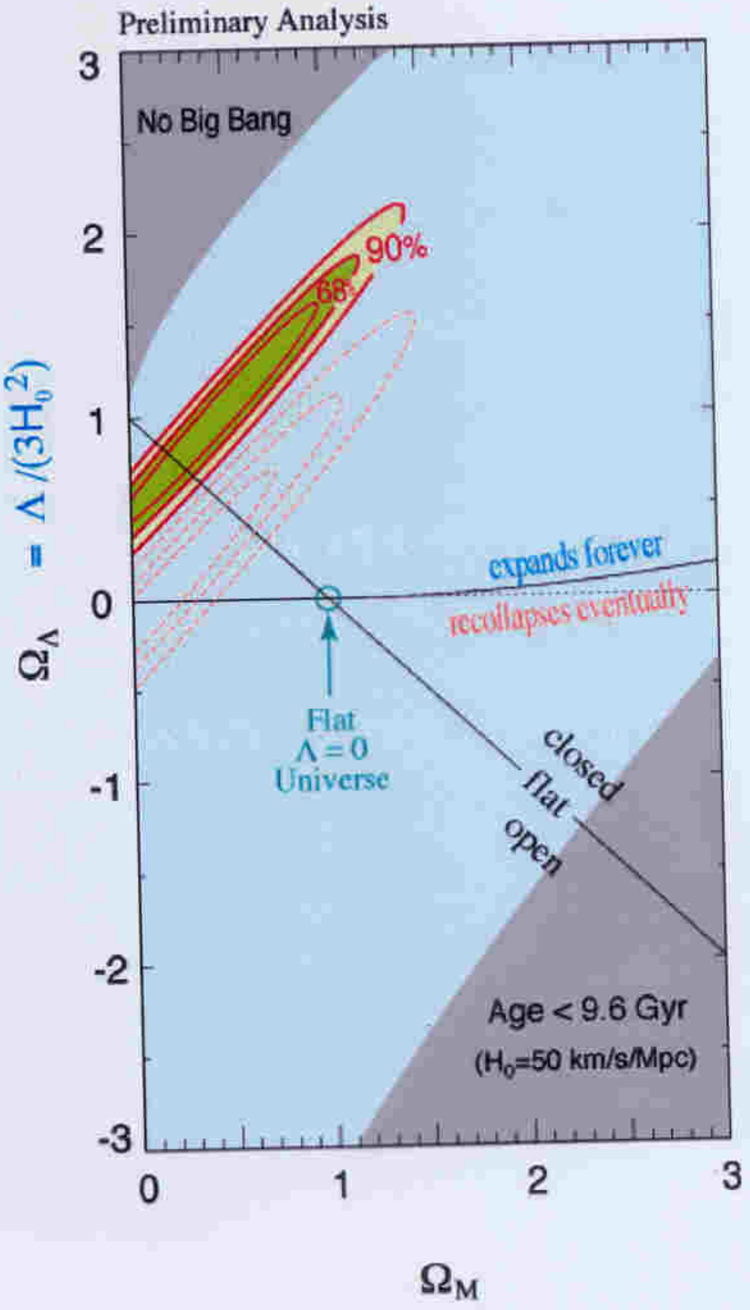


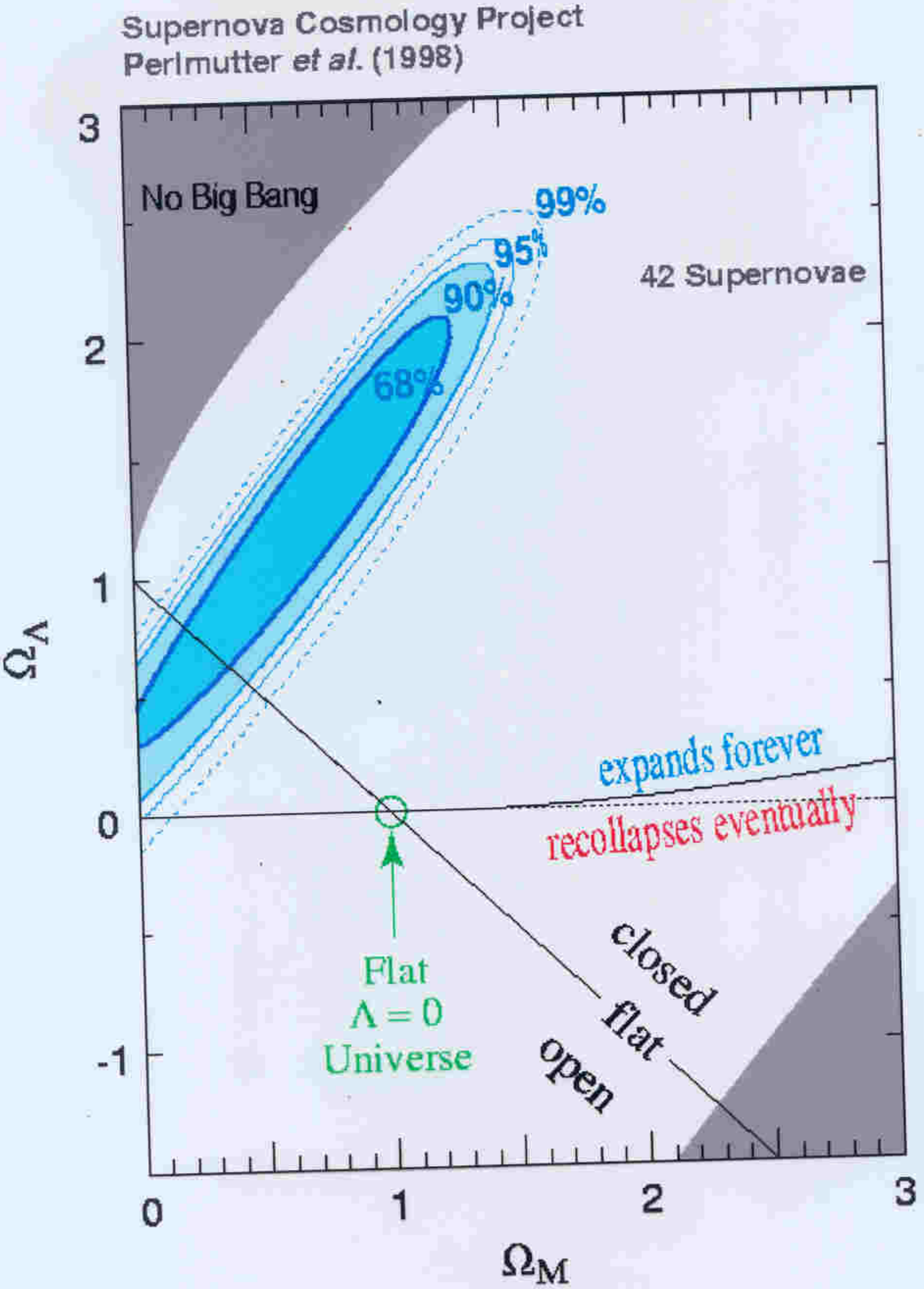
$(\Omega_M, \Omega_\Lambda) = (0, 1)$
(0.5,0.5) (0, 0)
(1, 0) (1, 0)
(1.5,-0.5) (2, 0)
Flat
$\Lambda = 0$
Supernova Cosmology Project
Calan/Tololo (Hamuy et al, A.J. 1996)
effective m_B
mag residual
$(\Omega_M, \Omega_\Lambda) =$
(0, 1)
(0.28, 0.72)
(0, 0)
(0.5, 0.5)
(0.75, 0.25)
(1, 0)
standard deviation
redshift z

Results: Ω vs Λ from 6 supernovae

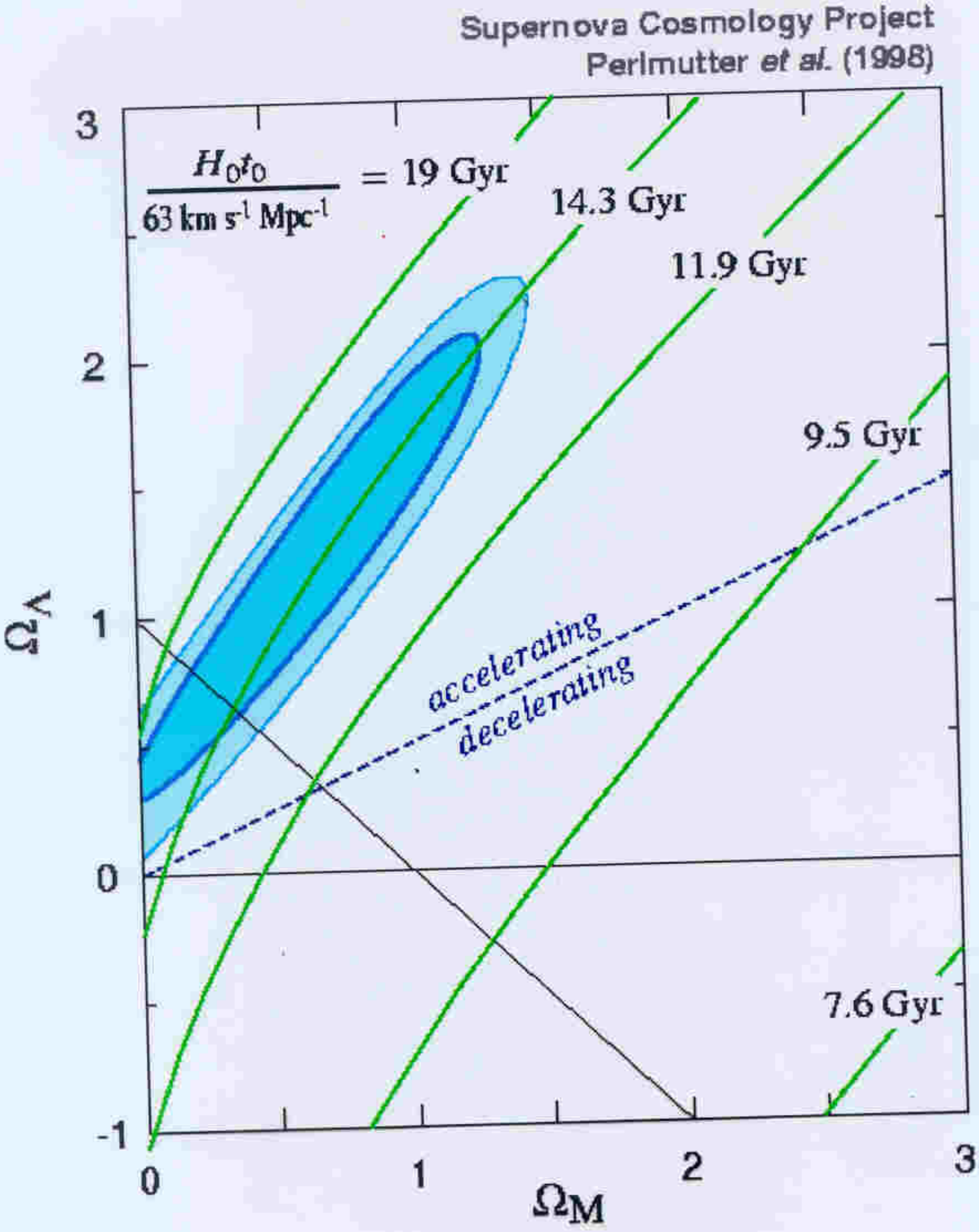


Results: Ω vs Λ from 40 supernovae



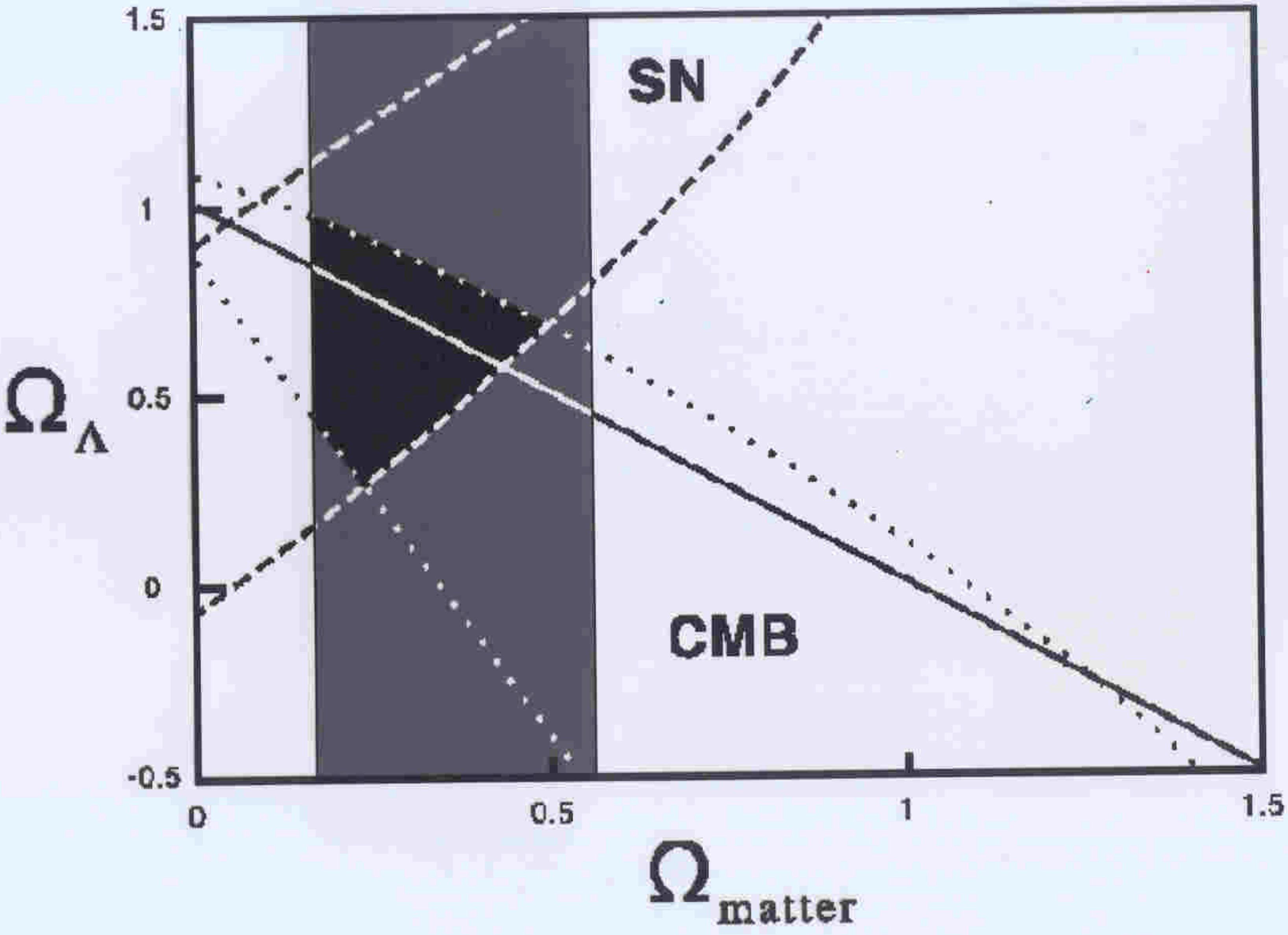


Supernova Cosmology Project
Perlmutter et al. (1998)
No Big Bang
99%
95%
90%
68%
42 Supernovae
expands forever
recollapses eventually
closed
flat
open
Flat
Λ = 0
Universe
3
2
1
0
-1
0
1
2
3
Ω_Λ
Ω_M

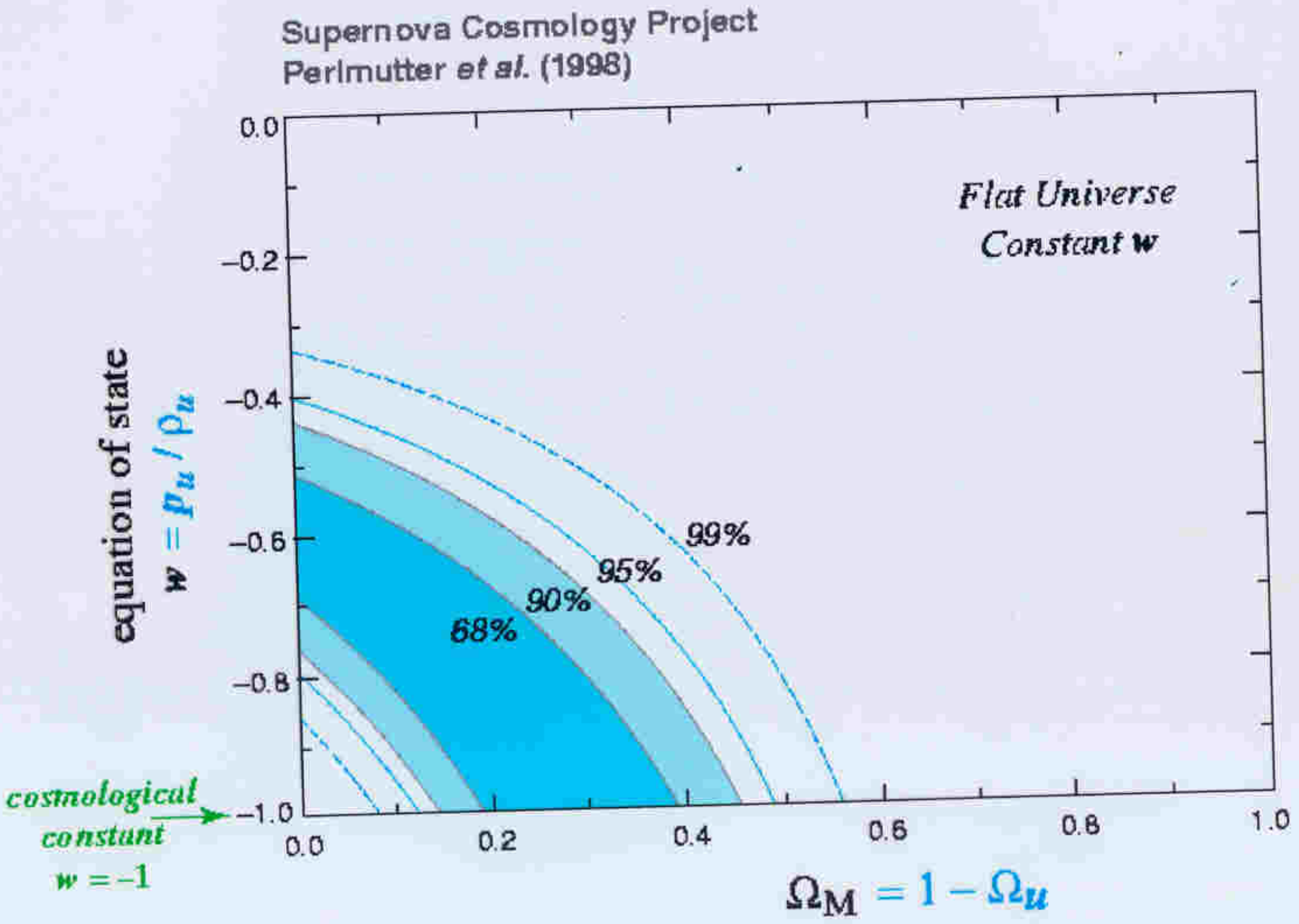


Best fit age of universe: $t_o = 14.5 \pm 1\ \ (0.63/h)$ Gyr

Best fit in flat universe: $t_o = 14.9 \pm 1\ \ (0.63/h)$ Gyr turner and tyson



Unknown Component Ω_u of Energy Density



turner and Tyson