THE ABSOLUTE COSMIC RAY MUON SPECTRUM AT SEA LEVEL

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The absolute vertical muon intensity in the range 0.2 - 1000 GeV/c has been determined from 4 different measurements. The best fit differential and integral intensities turned out to be 10 - 25% higher than previous spectra which have usually been normalized to the intensity at 1 GeV/c given by Rossi in 1948.

In most cosmic ray experiment, in which the muon spectra at sea level are being measured, the shape of the spectra rather than the absolute intensity has been of primary importance. It has been normal procedure to normalize the vertical differential spectrum to the value 2.45×10^{-3} $cm^{-2}sr^{-1}sec^{-1}(GeV/c)^{-1}$ at 1 GeV/c [1]. Recently however the intensity at this standard momentum has been remeasured [2, 3] and turned out to be 26% higher than the Rossi value. In consequence doubt has been cast on the validity of those spectra which were normalized to the Rossi value [4]. In the following an absolute spectrum in the range 0.2 - 1000 GeV/c is represented which was determined by four different measurements carried out in Kiel.

The measured absolute vertical intensities of three spark chamber spectrographs and one absorption spectrograph are presented in table 1. The smallest spectrograph recorded the flux in the momentum range 0.2 - 6.0 GeV/c (exp. No. I) [5, 6], the second spectrograph in the range 0.2 - 10 GeV/c (III) [7, 8] and the largest spectrograph in the range 10 - 1000 GeV/c (IV) [9, 10]. From all three spectrographs only data which correspond to a relative acceptance larger than 0.7 and a momentum smaller than the mdm of the instrument were accepted in table 1. By means of an absorption spectrograph the absolute vertical intensity has been determined at 1.11 GeV/c (II) respectively 1 GeV/c [2, 3]. The intensities of the first two spectrographs (I, III) have been measured in relative units because of the uncertaincy in the aperture, which is about 10%. These data were normalized to the absorption measurement at 1.11 GeV/c (II). The relative intensity errors (I, III) includes both statistical as normalization errors The data of IV were measured in absolute units. All data represent the total spectrum of muon irrespective of whether the muons are accompanied by secondaries or not.

The experimental data of table 1 were used to determine an absolute vertical muon spectrum at sea level. In a theoretical analysis of our data we have fitted our distribution to the following phenomenological form:

$$\frac{\mathrm{d}N(E)}{\mathrm{d}E} = PA\,(E + \Delta E)^{-\gamma}$$

$$\times\{r_1^{j-1} B_1 (E+\Delta E + B_1)^{-1} + K r_2^{\gamma-1} B_2 (E+\Delta E + B_2)^{-1}\}\,,$$

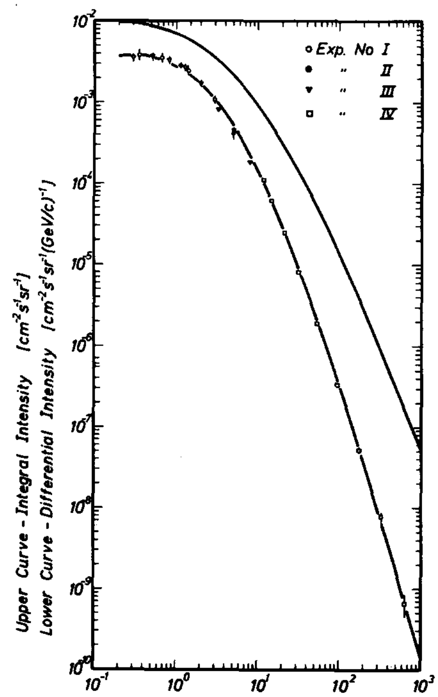

Fig. 1. The absolute vertical muon spectra in Kiel.

Table 1
The measured absolute vertical muon intensities in Kiel

Momentum range GeV/c	Mean momentum GeV/c	Differential intensity $cm^{-2}sr^{-1}sec^{-1}(GeV/c)^{-1}$	Relative error %	Number of meas. particles	Relative acceptance	Exp. No. ref.
0.2 - 0.5	0.34	$(3.92 \pm 0.62) \times 10^{-3}$	16.0	125	0.83	I
0.5 - 0.8	0.64	$(3.59 \pm 0.49) \times 10^{-3}$	13.5	121	0.91	[5,6]
0.8 - 2.0	1.32	$(2.57 \pm 0.29) \times 10^{-3}$	11.3	360	0.96	
2.0 - 4.0	2.86	$(1.11 \pm 0.13) \times 10^{-3}$	11.7	259	1.00	
4.0 - 6.0	4.91	$(4.13 \pm 0.59) \times 10^{-4}$	14.0	96	1.00	
0.98- 1.24	1.11	$(2.90 \pm 0.20) \times 10^{-3}$	6.8	6030	1.00	II [2,3]
0.2 - 0.4	0.29	$(3.57 \pm 0.35) \times 10^{-3}$	9.7	224	0.79	III
0.4 - 0.6	0.50	$(3.70 \pm 0.35) \times 10^{-3}$	9.4	270	0.92	[7,8]
0.6 - 1.0	0.79	$(3.41 \pm 0.28) \times 10^{-3}$	8.3	524	0.97	
1.0 - 1.5	1.24	$(2.73 \pm 0.23) \times 10^{-3}$	8.3	537	1.00	
1.5 - 2.5	1.95	$(1.73 \pm 0.15) \times 10^{-3}$	8.1	685	1.00	
2.5 - 4.0	3.17	$(7.92 \pm 0.65) \times 10^{-4}$	8.3	471	1.00	
4.0 - 6.0	4.91	$(4.24 \pm 0.37) \times 10^{-4}$	8.8	337	1.00	
6.0 - 10.0	7.76	$(1.84 \pm 0.17) \times 10^{-4}$	9.3	292	1.00	
10 - 13	11.4	$(1.13 \pm 0.01) \times 10^{-4}$	1.1	7670	0.73	IV
13 - 17	14.8	$(6.04 \pm 0.08) \times 10^{-5}$	1.2	6470	0.86	[9,10]
17 - 25	20.5	$(2.51 \pm 0.03) \times 10^{-5}$	1.3	5931	0.95	
25 - 40	31.4	$(8.01 \pm 0.13) \times 10^{-6}$	1.8	3754	1.00	
40 - 70	52.3	$(1.89 \pm 0.05) \times 10^{-6}$	2.4	1767	1.00	
70 - 128	93.0	$(3.38 \pm 0.14) \times 10^{-7}$	4.0	613	1.00	
128 - 250	175.0	$(5.19 \pm 0.37) \times 10^{-8}$	7.1	198	1.00	
250 - 450	329.0	$(7.84 \pm 1.12) \times 10^{-9}$	14.3	49	1.00	
450 - 1000	642.0	$(6.40 \pm 1.92) \times 10^{-10}$	30.2	11	1.00	

where B_1 = 90 GeV, B_2 = 450 GeV, r_1 = 0.76, r_2 = 0.52, E = muon energy at sea level, ΔE = energy loss from production to sea level, P = survival probability of muons from production to sea level. The unknown parameters are A, γ, K. A maximum likelihood technique has been used for the fitting. The best fit spectrum yields $A = 0.199$, $\gamma = 2.63$, $K = 0$. From these best fit parameters the differential and integral intensities at sea level and the corresponding slopes were calculated at standard momenta (table 2). In fig. 1 both spectra are presented together with the experimental data of table 1.

In fig. 2 the differential and integral spectra of other measurements were compared with the Kiel spectra (table 2). The Kiel spectrum has been used as reference spectrum, the error band is due to the errors of our data. The differential flux of the HW-spectrum [11] is lower by 10 - 25% in the momentum region 0.4 - 4.0 GeV/c. The spectral shapes of both spectra are similar for momenta larger than 10 GeV/c but they are different in the absolute intensity by 25%. Consequently the integral flux of the HW-spectrum is nearly of the same spectral shape, its amplitude however being lower by 20 - 25%. Regarding the OWP-spectrum [12] the relative difference is about 20% at 20 GeV/c and amounts to 2 - 10% in the region of 1 TeV. The relative difference on the spectrum of Menon et al. [13] is greatest (25%) at 20 GeV/c and is negligible for energies larger than 150 GeV/c. The absolute intensity

Table 2
The best fit absolute muon spectra and the corresponding power exponent at standard momenta

Momentum GeV/c	Differential intensity $\frac{cm^{-2}sr^{-1}sec^{-1}}{GeV/c}$	γ_{diff}	Integral intensity $cm^{-2}sr^{-1}sec^{-1}$	γ_{int}
0.2	3.73×10^{-3}	0.06	9.94×10^{-3}	0.05
0.4	3.72×10^{-3}	0.16	9.18×10^{-3}	0.13
0.8	3.10×10^{-3}	0.38	7.81×10^{-3}	0.28
1.0	2.79×10^{-3}	0.49	7.22×10^{-3}	0.35
1.5	2.14×10^{-3}	0.73	6.00×10^{-3}	0.50
2.0	1.67×10^{-3}	0.93	5.05×10^{-3}	0.63
3.0	1.06×10^{-3}	1.24	3.72×10^{-3}	0.82
5.0	4.97×10^{-4}	1.63	2.26×10^{-3}	1.08
7.0	2.73×10^{-4}	1.87	1.52×10^{-3}	1.24
10	1.33×10^{-4}	2.10	9.42×10^{-4}	1.40
15	5.40×10^{-5}	2.32	5.13×10^{-4}	1.57
20	2.70×10^{-5}	2.46	3.21×10^{-4}	1.68
30	9.59×10^{-6}	2.63	1.57×10^{-4}	1.82
50	2.36×10^{-6}	2.83	5.93×10^{-5}	1.99
70	8.92×10^{-7}	2.95	2.98×10^{-5}	2.09
100	3.04×10^{-8}	3.07	1.38×10^{-5}	2.20
150	8.51×10^{-8}	3.20	5.55×10^{-6}	2.30
200	3.35×10^{-8}	3.28	2.84×10^{-6}	2.36
300	8.70×10^{-9}	3.37	1.07×10^{-6}	2.43
500	1.52×10^{-9}	3.46	3.03×10^{-7}	2.50
700	4.71×10^{-10}	3.50	1.30×10^{-7}	2.54
1000	1.34×10^{-10}	3.54	5.23×10^{-8}	2.56

measurement with the MARS spectrograph of Wolfendale [14] are indicated by open circles and lower than the reference spectrum by 11% and 17% respectively. The measured intensity of Sheldon [15] is lower by 5 - 15% than the reference spectrum. The results of the measurements of Cottrell et al. [16] have been reanalysed in terms of their energy dependence and are plotted as triangles in fig. 2. At 5 GeV/c their intensity is higher by 40% than the reference spectrum, the other two points correspond to the HW-line. The spectrum of Aurela et al. [17] is lower by 20% in the intervall 1 - 100 GeV/c and in agreement with the present work for momenta greater than 100 GeV/c.

The present work indicates that the absolute intensity spectra of muons were underestimated by 10 - 20% by many investigators.

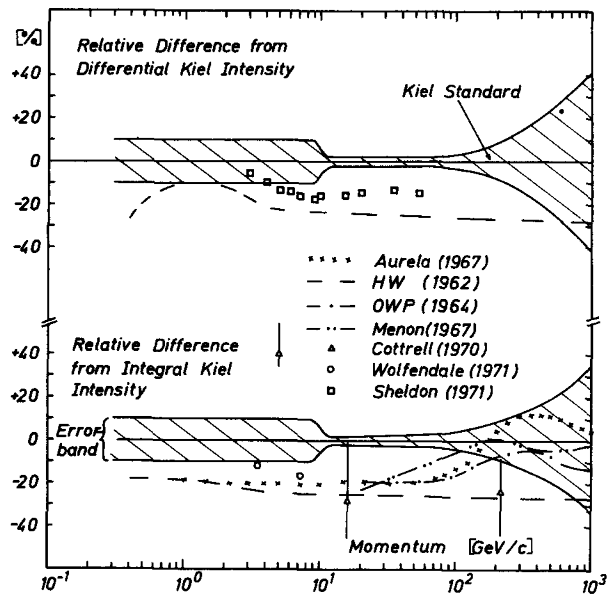

Fig. 2. Relative differences of other workers from the best fit Kiel intensity e.g. (diff Int_{HW} - diff Int_{Kiel})/diff Int_{Kiel}.

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