New COMPASS results on $A_1^p$ and $g_1^p$ and QCD fit

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Introduction
Decomposition

\[ S = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + \begin{cases} L_q + L_{\bar{q}} \end{cases} \]

orbital angular momenta

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

\[ \Delta q \equiv \Delta (q + \bar{q}) \]
\[ \Delta q = \vec{q} - \vec{\bar{q}} \] (parallel minus antiparallel to the nucleon spin)
\[ g_1(x, Q^2) \simeq \sum_q e_q^2 \Delta q(x, Q^2) \]

"Spin crisis"

- Relativistic quark model prediction: \( \Delta \Sigma \simeq 0.6 \)
- SMC measurement (1988): \( \Delta \Sigma = 0.12 \pm 0.17 \)

Recent status

- Quark spin contributes only about 30\% to the nucleon spin
- Gluon contribution constrained only for a limited \( x \) range
- Very few experimental results on orbital angular momentum
COMPASS @ CERN

**COMPASS experiment**

**COmmom Muon P**roton **A**pparatus for **S**tructure and **S**pectroscopy

- **Fixed target experiment** at the SPS using a tertiary muon beam
- Collaboration of about 200 members from 11 countries and 23 institutions

**160/200 GeV $\mu^+$ polarised beam**, $P_b \sim -80\%$

- **$^6\text{LiD}$ or NH$_3$, 1.2 m long, polarised target @ 2.5 T and 60 mK**, $P_{\text{target}} \sim 50/85\%$

- large acceptance, two staged spectrometer
- tracking, calorimetry, RICH

$A^D_1$ and $g^D_1$ and QCD fit (COMPASS)

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DIS and spin observables

**Experimental asymmetry**

\[ A_{\text{exp}} = \frac{N\rightarrow - N\rightarrow}{N\rightarrow + N\rightarrow} = P_{\text{beam}} P_{\text{target}} f A_{\parallel} \]

**Lepton-nucleon asymmetry**

\[ A_{\parallel} = \frac{d\sigma \rightarrow - d\sigma \rightarrow}{d\sigma \rightarrow + d\sigma \rightarrow} \simeq DA_1 \quad A_1 \simeq A_{\parallel}/D \]

**Virtual photon-nucleon asymmetry**

\[ A_1 = A_1^{\gamma^* N} = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \simeq \frac{g_1}{F_1} \]

**Spin dependent structure function \( g_1 \)**

\[ g_1(x, Q^2) \simeq \frac{F_2(x, Q^2)}{2x(1 + R(x, Q^2))} A_1(x, Q^2), \quad \text{with} \quad R \equiv \frac{\sigma_L}{\sigma_T} \]

\[ k_\mu = (E_\mu, k_\mu) \]
\[ k'_\mu = (E'_\mu, k'_\mu) \]
\[ P = (M, 0) \]
\[ q = k_\mu - k'_\mu = (\nu, q) \]
\[ Q^2 = -q^2 \]
\[ \nu = P \cdot q/M = E_\mu - E'_\mu \]
\[ W^2 = M^2 + 2M\nu - Q^2 \]
\[ x = Q^2/(2M\nu) \]
\[ y = \nu/E_\mu \]
New results on $A_1^p$ and $g_1^p$ for $Q^2 > 1 \text{ GeV}^2/c^2$
Data taken by COMPASS in 2007 @ 160 GeV/c and in 2011 @ 200 GeV/c

Obtained giving each event a weight \( \omega = f |D|P_b | \) to optimize the statistical errors of the results

Unpolarised radiative corrections (RC), included in the dilution factor, from TERAD\(^1\)

Polarised radiative corrections from POLRAD\(^2\)

Corrected for polarisable \(^{14}\)N in the ammonia target
Results on $A_1^p$ in DIS ($Q^2 > 1$ GeV$^2$/c$^2$)

- New asymmetries at low $x$
- Results from two beam energies compatible
- Well fit by constant

$A_1^p$ and $g_1^p$ and QCD fit (COMPASS)  
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Results on $g_1^d$ and $g_1^p$ in DIS ($Q^2 > 1\,\text{GeV}^2/c^2$)

- New COMPASS point for the proton at low $x$
- New COMPASS NLO QCD fit describes the data well
139 out of 674 points are from COMPASS

\[ g_1^p(n) = \frac{1}{9} \left[ C_S \otimes \Delta q_S + C_{NS} \otimes \left( \pm \frac{3}{4} \Delta q_3 + \frac{1}{4} \Delta q_8 \right) + C_g \otimes \Delta g \right] \]

- \( \Delta q_S = \Delta u + \Delta d + \Delta s \) (spin singlet parton distribution)
- \( \Delta q_3 = \Delta u - \Delta d \) (triplet non-singlet spin distribution)
- \( \Delta q_8 = \Delta u + \Delta d - 2\Delta s \) (octet non-singlet spin distribution)
- \( C_S, C_{NS}, C_g \): Wilson coefficients associated to each distribution

Functional forms are assumed at a given reference scale \( Q_0^2 \)

\( SU(3)_f \) to fix the non-singlet distributions first moments:

\[ \int_0^1 (\Delta u - \Delta d) \, dx = F + D = g_A/g_V \quad \text{and} \quad \int_0^1 (\Delta u + \Delta d - 2\Delta s) \, dx = 3F - D \]

Positivity: \( |\Delta g(x)| < |g(x)| \) and \( |\Delta(s(x) + \bar{s}(x))| < |s(x) + \bar{s}(x)| \)
NLO QCD fit results

- Depending upon assumed functional forms, **3 categories of solutions**: \( \Delta G > 0, \Delta G \sim 0 \) and \( \Delta G < 0 \)

- Gluon polarisation: \( \Delta G \) **not well constrained by the fit**
  \( \leftrightarrow \) direct measurements needed

- Quark polarisation: \( 0.26 < \Delta \Sigma < 0.34 \) at \( Q^2_0 = 3 \text{ (GeV/c)}^2 \ (\overline{\text{MS}}) \)
  \( \leftrightarrow \) largest uncertainty from functional forms

- Large uncertainty at very low \( x \) for \( g_1^p \) and \( g_1^d \)
Test of the Bjorken sum rule

\[ \int_0^1 g_{1NS}^N(x, Q^2) \, dx = \int_0^1 \left[ g_1^p(x, Q^2) - g_1^n(x, Q^2) \right] \, dx = \frac{1}{6} \left| \frac{g_A}{g_V} \right| C_{1NS}^N(Q^2) \]

- Fundamental QCD prediction connecting p and n
- Test of SU(2)\text{flavour}
- Decorrelated from $\Delta G$
- $g_{1NS}$ from COMPASS data alone (w/ proton and deuteron targets):

\[ g_{1NS} = g_1^p - g_1^n = 2 \left[ g_1^p - \frac{g_1^d}{1 - 3/2 \cdot \omega_D} \right], \text{ with } \omega_D = 0.05 \pm 0.01 \]

\[ C_{1NS}^N = 1_{LO} - \left( \frac{\alpha_S}{\pi} \right)_{NLO} - p_1 \left( \frac{\alpha_S}{\pi} \right)_2^{NLO} - p_2 \left( \frac{\alpha_S}{\pi} \right)_3^{NNLO} - \ldots \]

\[ \left| \frac{g_A}{g_V} \right| = 1.2701 \pm 0.0020 \text{ (from neutron } \beta \text{ decay)} \]
Results on the Bjorken sum rule

$\Gamma^{NS}_{1}(Q^2 = 3 GeV^2/c^2) = 0.181 \pm 0.008^{\text{(stat)}} \pm 0.014^{\text{(syst)}}$

$\left| \frac{g_A}{g_V}\right| = 1.2701 \pm 0.0020$ (from neutron $\beta$ decay)

$\left| \frac{g_A}{g_V}\right| = 1.220 \pm 0.053^{\text{(stat)}} \pm 0.095^{\text{(syst)}}$ using $C^{NS}_1@\text{NLO}$

$\left| \frac{g_A}{g_V}\right| = 1.256 \pm 0.054^{\text{(stat)}} \pm 0.098^{\text{(syst)}}$ using $C^{NS}_1@\text{NNLO}$

Bjorken sum rule validated within 4%
New results on $A_1^p$ and $g_1^p$ for $Q^2 < 1 \text{ GeV}^2/c^2$
Motivation for the low $x$, low $Q^2$ studies

- Low $x \Leftrightarrow$ **high parton densities**

- Fixed target experiments $\Leftrightarrow$ **strong correlation** between $x$ and $Q^2$: low $x \Rightarrow$ low $Q^2$, where pQCD isn’t expected to work

- Some **models, to be confronted with data**, allow a smooth extrapolation to the low-$Q^2$ and high-$Q^2$ **regions** (resummation, VMD):
  
  B. Badelek et al, B.I. Ermolaev et al.

- $A_1^p$ and $g_1^p$ at low $x$ and low $Q^2$:
  - can be measured with **improved precision**
  - complement our measurement of $A_1^d$ and $g_1^d$ at low $x$ and low $Q^2$
  - $g_1^{NS} = g_1^p - g_1^n$ can be extracted
  - also as functions of $\nu$, as suggested by theoreticians
Data samples for extraction of $A_1^p$ and $g_1^p$ $Q^2 < 1 \text{ GeV}^2/c^2$

- Data taken in 2007 & 2011 with a NH$_3$ target
- $676 \times 10^6$ events ($150 \times$ more than SMC)

**Main event selection criteria:**
- at least one additional track (besides the scattered muon) in the interaction vertex
- not a $\mu e$ elastic scattering event
- $Q^2 < 1 \text{ (GeV/c)}^2$
- $x \geq 4 \times 10^{-5}$
- $0.1 < y < 0.9$

$\langle Q^2 \rangle \ (\text{GeV}^2/c^2)$ vs $\langle x \rangle$

$\langle x \rangle$ vs $\langle \nu \rangle \ (\text{GeV})$

$W \ (\text{GeV})$
First COMPASS results for $A_1^p$ at low $x$ and low $Q^2$

- Procedure similar to the one for $Q^2 > 1 \text{ (GeV/c)}^2$ (weighting, radiative corrections, $^{14}\text{N}$ correction)

- The results for the two beam energies are compatible within errors.
- The systematic errors are smaller than the statistical errors (not shown here).
- A **significantly positive asymmetry** is observed.
- No significant dependence with $\nu$ is seen.
First COMPASS results for $g_1^p$ at low $x$ and low $Q^2$

- The structure function is obtained in bins of $x$ or $\nu$ according to:

$$g_1^p \left( \langle x \rangle, \langle Q^2 \rangle \right) = \frac{F_2^p \left( \langle x \rangle, \langle Q^2 \rangle \right)}{2x \left[ 1 + R \left( \langle x \rangle, \langle Q^2 \rangle \right) \right]} A_1^p \left( \langle x \rangle, \langle Q^2 \rangle \right)$$

- $F_2^p(\langle x \rangle, \langle Q^2 \rangle)$ from the SMC fit on data or from a model (for low $x$ and $Q^2$) \[3\]
- $R(\langle x \rangle, \langle Q^2 \rangle)$ based on SLAC parameterization, extended to low $Q^2$ \[4\]

The results for the two beam energies are compatible within errors.

The systematic errors are smaller than the statistical errors (not shown here).

$g_1^p$ is significantly positive.

No significant dependence with $\nu$ is seen.
The COMPASS results significantly improve the precision of the measurement.

Comparing with B. Badelek et al. [Phys. Rev. D 61 (1999) 014009], the COMPASS data favour \( C \in [0, +4] \), i.e. a VMD contribution to \( g_1 \) of the same sign of the partonic contribution.
Summary and outlook
Summary and outlook

- \( Q^2 > 1 \ (\text{GeV/c})^2 \):
  - New measurements of \( A_1^p \) and \( g_1^p \) at 200 GeV/c
  - New value at low \( x \), overall improved precision
  - Updated NLO QCD fit
  - Bjorken sum rule verified more accurately

- \( Q^2 < 1 \ (\text{GeV/c})^2 \):
  - First COMPASS results on \( A_1^p \) and \( g_1^p \) for \( Q^2 \in [0.001, 1] \ (\text{GeV/c})^2 \), \( x \in [4 \cdot 10^{-5}, 4 \cdot 10^{-2}] \), and \( \nu \in [14, 194] \) GeV, in bins of \( x \) or in bins of \( \nu \)
  - Results of \( A_1^p \) and \( g_1^p \) are significantly positive
  - \( A_1^p(x) \) results are compatible with the model of Badelek et al. (1999) for \( C \in [0, +4] \), i.e. they favour a VMD contribution to \( g_1 \) of the same sign as the partonic one

Next:

- \( Q^2 > 1 \ (\text{GeV/c})^2 \):
  - \( A_{1,p}^{\pi^\pm} \) and \( A_{1,p}^{K^\pm} \), polarised PDFs for each flavour

- \( Q^2 < 1 \ (\text{GeV/c})^2 \):
  - \( A_1^p \) and \( g_1^p \) in 2D bins, \( g_1^{NS} \) from \( g_1^p \) and \( g_1^d \)


Backup
139 out of 674 points are from COMPASS

\[ g_1^{p(n)} = \frac{1}{9} \left[ C_S \otimes \Delta q_S + C_{NS} \otimes \left( \pm \frac{3}{4} \Delta q_3 + \frac{1}{4} \Delta q_8 \right) + C_g \otimes \Delta g \right] \]

- \( \Delta q_S = \Delta u + \Delta d + \Delta s \) (spin singlet parton distribution)
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- \( \Delta q_8 = \Delta u + \Delta d - 2\Delta s \) (octet non-singlet spin distribution)
- \( C_S, C_{NS}, C_g \): Wilson coefficients associated to each distribution

Functional forms at a given reference scale \( Q_0^2 \):
- \( \Delta q_S(x, Q_0^2) = \eta_S x^{\alpha_S} (1 - x)^{\beta_S} (1 + \gamma_S + \rho_S \sqrt{x}) / N_S \)
- \( \Delta q_g(x, Q_0^2) = \eta_g x^{\alpha_g} (1 - x)^{\beta_g} (1 + \gamma_g + \rho_g \sqrt{x}) / N_g \)
- \( \Delta q_3(x, Q_0^2) = \eta_3 x^{\alpha_3} (1 - x)^{\beta_3} / N_3 \)
- \( \Delta q_8(x, Q_0^2) = \eta_8 x^{\alpha_8} (1 - x)^{\beta_8} / N_8 \)
Inputs and constraints to the NLO QCD fit

- $SU(3)_f$ to fix the non-singlet distributions first moments:
  
  \[
  \int_0^1 (\Delta u - \Delta d) \, dx = F + D = g_A/g_V
  \]

  \[
  \int_0^1 (\Delta u + \Delta d - 2\Delta s) \, dx = 3F - D
  \]

  $F, D$: parameters describing the weak axial-vector/vector coupling constants

- $\beta_g$ is fixed

Evolution using DGLAP equations

Minimize:

\[
\chi^2 = \sum_{n=1}^{N_{\text{exp}}} \left[ \sum_{i=1}^{N_{\text{data}}} \left( \frac{g_{1,i}^{\text{fit}} - N_n g_{1,i}^{\text{data}}}{N_n \sigma_i} \right)^2 + \left( \frac{1 - N_n}{\delta N_n} \right)^2 \right] + \chi^2_{\text{positivity}}
\]

- Positivity: $|\Delta g(x)| < |g(x)|$ and $|\Delta(s(x) + \bar{s}(x))| < |s(x) + \bar{s}(x)|$

- Unpolarised PDFs for the positivity constraint: MSTW2008

In total: 28 free parameters and 679 data points
Influence of the input scale $Q_0^2$

$\Delta G$ with a node

$\Delta G < 0$

$\Delta G > 0$
Error band associated to statistical uncertainties

COMPASS Preliminary

$Q^2 = 3 \text{ (GeV/c)}^2$

- Results of 2,000 fits to the replicas
- Color: density of replicas at a given $x$
- Black curves: the border of the interval at 68% CL
Fit results

\[ \Delta G \text{ vs } \Delta \Sigma \]

COMPASS Preliminary

\[ Q^2 = 1 \text{ (GeV/c)}^2 \]

\[ \Delta \chi^2 = 1 \text{ (39.3\%)} \]
\[ \Delta \chi^2 = 2.41 \text{ (70\%)} \]
\[ \Delta \chi^2 = 4.61 \text{ (90\%)} \]
Calculate $\int_0^1 g_1^{NS} \, dx$

- Calculate $g_1^{NS}$
- Perform a NLO QCD fit, fitting only $\Delta q_3$ (3 parameters needed) @ $Q_0 = 1$ (GeV/c)$^2$
- Evolve $g_1^{NS}$ to $Q_0 = 3$ (GeV/c)$^2$
- Use extrapolation to $x \to 0$ and to $x \to 1$ (94% of $\int_0^1 g_1^{NS} \, dx$ is in the measured range)
Model details


(VMD contribution and QCD improved parton model extended to low $Q^2$)

\[ g_1(x,Q^2) = g_{1}^{VMD}(x,Q^2) + g_{1}^{part}(x,Q^2). \]  \hspace{1cm} (4)

\[ g_{1}^{VMD}(x,Q^2) = \frac{pq}{4\pi} \sum_{\nu=\rho,\omega,\phi} \frac{m_{\nu}^4 \Delta \sigma_{\nu}(W^2)}{\gamma_{\nu}^2(Q^2+m_{\nu}^2)^2}. \]  \hspace{1cm} (5)

\[ \Delta \sigma_{\nu} = \frac{\sigma_{1/2} - \sigma_{3/2}}{2}, \]  \hspace{1cm} (6)

\[ \frac{pq}{4\pi} \sum_{\nu=\rho,\omega} \frac{m_{\nu}^4 \Delta \sigma_{\nu}}{\gamma_{\nu}^2(Q^2+m_{\nu}^2)^2} \]

\[ = C \left[ \frac{4}{9} (\Delta u_\nu^0(x) + 2 \Delta \bar{u}^0(x)) + \frac{1}{9} (\Delta d_\nu^0(x) + 2 \Delta \bar{d}^0(x)) \right] \]

\[ \times \frac{m_{\rho}^4}{(Q^2+m_{\rho}^2)^2}, \]  \hspace{1cm} (7)

\[ \frac{pq}{4\pi} \frac{m_{\phi}^4 \Delta \sigma_{\phi}}{\gamma_{\phi}^2(Q^2+m_{\phi}^2)^2} = C \frac{2}{9} \Delta s_\phi^0(x) \frac{m_{\phi}^4}{(Q^2+m_{\phi}^2)^2}, \]  \hspace{1cm} (8)

\[ \Delta p_{j}^0(x) = C_{j}(1-x)^{\eta_{j}}. \]  \hspace{1cm} (3)