Percolation and cosmic rays above 10^{17} eV

J. Alvarez-Muñiz*, P. Brogueira†, R. Conceição†, J. Dias de Deus†, M.C. Espírito Santo†, M. Pimenta†

*IGFAE and Dep. Fisica Particulas, Univ. Santiago de Compostela, 15782 Santiago de Compostela, Spain
†LIP-CENTRA-IST, Av. Elias Garcia, 14-1, 1000-149 Lisboa, Portugal

Abstract— In this work we argue that, in the interpretation of the energy dependence of the depth of the shower maximum and of the muon content in high energy cosmic ray showers ($E \gtrsim 10^{17}$ eV), other variables besides the composition may play an important role, in particular those characterising the first (high energy) hadronic collisions. The role of the inelasticity, of the nature of the leading particle, and of the particle multiplicity are discussed. A consistent interpretation of existing data within a string percolation model implemented in a hybrid, one dimensional simulation method is given.

I. INTRODUCTION

The composition of cosmic rays at high energies is since long a matter of controversy. Direct measurements are possible only up to $E \sim 10^{15}$ eV [1]. Above these energies, attempts to infer the effective mass number A of the primary particle are based on the measured shower variables and are rather indirect. Furthermore, data are relatively scarce at such high energies. This is in fact a long standing question, and the variables used as experimental handles in composition have been, themselves, proposed decades ago [2], [3]. The study of the depth of the shower maximum is a widely used method for the derivation of cosmic ray composition [4], [5]. In fluorescence experiments, the longitudinal shower profile is measured and the atmospheric depth of the shower maximum X_{max} is usually interpreted as an indication of the nature of the primary particle [5]: heavy nuclei initiated showers are expected to develop earlier, due to the larger interaction cross-section. In ground array experiments, the muon component has been since long studied as a possible handle on composition [3], [5]. The muon content, or more specifically the muon density at a given distance from the shower core, ρ_μ, is expected to increase by roughly 50% from proton to Fe initiated showers, for the same primary energy [6]. Incidentally, the muon component of the shower is a powerful tool for the validation of hadronic interaction models at such high energies. In this work we argue that, in the interpretation of the energy dependence of X_{max} and N_μ, the number of muons in the shower, other variables besides A may play an important role, in particular those characterising the first (high energy) hadronic collisions. The role of the inelasticity K, one minus the fraction of momentum carried by the fastest (leading) particle, of the nature of the leading particle, and of the charged particle multiplicity are discussed. A consistent interpretation of data, within a string percolation model implemented in a hybrid one dimensional simulation method, is given. More details can be found in [7].

II. EXPERIMENTAL STATUS

Experimental data on the energy dependence of the depth of the shower maximum X_{max} and of the muon density 600 m away from the shower core $\rho_\mu(600)$ above 10^{17} eV, and their comparison with simulations, are summarised in this section. Fly's Eye/HiRes results in this energy range show an increase in the X_{max} vs. E slope, which has been interpreted as a change in composition, going from more Fe-like to more proton-like showers [8]. It has been argued [9], [10] that other models predicting a development of the shower deeper in the atmosphere (an increase in X_{max}) could also explain this effect. Concerning the muon component of the shower, experimental data are relatively scarce at high energies. measurements have been done in KASCADE [11] in the energy range from 10^{14} eV to almost 10^{17} eV and in AGASA above 10^{17} eV [12]. Hybrid data from HiRes-MIA has also been presented [13]. At AGASA, the lateral distribution function of muons above 0.5 GeV was measured and combined with the Akeno 1 Km² array data (threshold 1 GeV) [12]. The evolution of $\rho_\mu(600)$ with the particle density $S_0(600)$ (essentially proportional to the primary energy) was studied and compared with CORSIKA [14] simulations using different hadronic models in [6]. It was observed that the slope of $\rho_\mu(600)$ vs. E in data is flatter than in simulation with any hadronic model and primary composition. Taking $\rho_\mu \propto E^\beta$, data gives $\beta = 0.84 \pm 0.02$ [6], [12], while from simulation $\beta \sim 0.9$ ($\beta = 0.92$ (0.89) for protons with QGSJET (SIBYLL), while for Fe $\beta = 0.88$ (0.87) [6]). It has been argued [15] that these results can be interpreted as a change in composition from heavy (at around $10^{17.5}$ eV) to light (at around 10^{19} eV), agreeing with the Fly's Eye indication. It has been pointed out in [6] that this interpretation should be studied in a wider energy range, as it seems to lead to a composition heavier than Fe at lower energies. The HiRes-Mia experiment consisted of the prototype high resolution Fly's Eye detector and the muon array MIA. Results are presented in [13]. The muon data, yielding $\beta = 0.73$, seem to indicate a composition heavier than Fe at 10^{17} eV.

III. CONSTRUCTION OF A SIMPLE MODEL

A. The composition interpretation

In order to make the argument simple, we shall use the original Heitler idea [16]: the location of the shower maximum is, on the average, related to $\log E$, $\bar{X}_{max} \sim \log E$, and the number of charged pions or muons is proportional to some power of E, $N_{\pi^+\pi^-} \sim N_\mu \sim E^\beta, 0 < \beta \lesssim 1$. In the case of a

nucleus with A nucleons colliding in a hadronic collision we shall write,

$$\bar{X}_{max} \simeq \bar{X}_1 + \bar{X}_0 \log(E/A), \tag{1}$$

where $\bar{X}_1$ is the average depth of the first collision and $\bar{X}_0$ is a model dependent parameter given by the elongation rate [17] (for electromagnetic showers, $\bar{X}_0$ is the radiation length), and

$$\bar{N}_\mu \simeq A(E/A)^\beta, \tag{2}$$

or

$$\log N_\mu \simeq (1 - \beta) \log A + \beta \log E . \tag{3}$$

We further have, for the elongation rate,

$$\frac{d\bar{X}_{max}}{d\log E} = \bar{X}_0 \left[1 - \frac{d\log A}{d\log E}\right], \tag{4}$$

and for the $\log N_\mu$ dependence on E,

$$\frac{d\log N_\mu}{d\log E} = (1 - \beta)\frac{d\log A}{d\log E} + \beta . \tag{5}$$

As experimentally [8], [13], [18], above 10^{17} eV $d\bar{X}_{max}/d\log E$ is larger and $d\log N_\mu/\ d\log E$ is slightly smaller, in comparison with lower energies, the conclusion is:

$$\frac{d\log A}{d\log E} < 0 , \tag{6}$$

i.e., for $E \gtrsim 10^{17}$ eV the average mass number A should, in this interpretation, decrease with energy.

B. The first hadronic collisions interpretation

In this interpretation a key role is given to the main variables characterising the first hadronic collisions. The inelasticity K is the fraction of energy distributed among the produced particles (mainly pions). A fraction of this energy is expected to go into neutral pions, which promptly decay to photons. $< n >$ is the average (non-leading) multiplicity at the collision energy. The quantity $(1 - K)$ represents the fraction of energy concentrated in the leading particle, in general assumed to be a proton. P_0 is the probability of having a leading π^0. As pointed out in [9], [19], the effect of changing A is equivalent to the effect of changing the average inelasticity K. In this spirit, the fastest particle, carrying an energy $(1 - K)E$, will originate the shower branches that go deeper in the atmosphere, and we write

$$\bar{X}_{max} \simeq \bar{X}_1 + \bar{X}_0 \log[(1 - K)\frac{E}{E_0}] , \tag{7}$$

instead of (1), where $\bar{X}_1$ is the average depth of the first collision, $\bar{X}_0$ is the elongation rate and E_0 a low energy threshold. Regarding the muon content of the shower, a possible assumption is to say that while the energy flows in the $(1 - K)$ direction the number of particles (of muons) flows in the K direction. We thus have

$$N_\mu \sim N_\pi^\pm \propto KE . \tag{8}$$

instead of (2). The validity of this assumption will be discussed later. We further have,

$$\frac{d\bar{X}_{max}}{d\log E} = \bar{X}_0 \left[\frac{d\log(1 - K)}{d\log E} + 1\right] , \tag{9}$$

instead of (4), and

$$\frac{d\log N_\mu}{d\log E} = \frac{d\log K}{d\log E} + 1 , \tag{10}$$

instead of (5). The condition (6) becomes now

$$\frac{d\log K}{d\log E} < 0 , \tag{11}$$

i.e., the inelasticity K has to decrease with the energy. This scenario has been explored in [10], where a string percolation model predicting the required behaviour of the inelasticity at high energies was discussed. As it will be seen below, string percolation affects the energy dependence of K, P_0 and $< n >$.

C. Matthews-Heitler toy model

In [20] a very interesting semi-empirical model for the development of the hadronic component of air showers in analogy with Heitler's splitting approximation of electromagnetic cascades [16] was presented. Not considering the elasticity, the model gives constant and reasonable values for $dX_{max}/d\log E$ and $d\log N_\mu/d\log E$. Taking into account that in hadronic interactions a significant fraction of the total energy may be carried away by a single "leading" particle (i.e., $1 - K > 0$), the model predicts for N_μ a slope decreasing with K - exactly the opposite of (8). It is interesting to discuss this discrepancy. In [20] the essential point is that the production of π^0's, which decay into photons before participating in the cascade, strongly decreases the muon production - about $1/3$ of the energy KE is lost for muon production, as it goes into the electromagnetic component of the shower. On the other hand, the leading particle, assumed in general to be a proton, goes on producing particles, thus increasing the number of muons. However, if π^0's are themselves leading particles - $P_0 > 0$, as it happens in percolation models, see section III-F - the argument is no longer valid. In fact, when estimating muon production both the inelasticity K and the probability P_0 of having a π^0 as a leading particle are relevant. This will be seen next.

D. Generalised Matthews-Heitler model

The model of [20] was implemented and generalised. This is not a substitute for a detailed simulation, but useful to illustrate the physics involved. Following [20], hadrons interact after traversing one layer of fixed thickness (related to the interaction length, $\lambda_I \sim 120$ g cm^2 [20], [21]) producing N_{ch} charged pions and $N_{ch}/2$ neutral pions. A π^0 immediately decays into photons, meaning that $1/3$ of the energy is lost to the electromagnetic shower component. Charged pions continue through another layer and interact, until a critical energy is reached. They are then assumed to decay, yielding muons, and thus $N_\mu = N_{\pi^\pm}$. The version of the model which takes into account the inelasticity was implemented. In each branching, the energy was equally shared amongst all (non-leading) particles. $N_{ch} = 10$ was used. We modified this model in two ways. Firstly, a decay probability for charged pions in each interaction step was explicitly included,

replacing the large value of the critical energy (20 GeV) used in the original version. This gives a smoother and more realistic transition from charged pions to muons. A critical energy of 1 GeV was then used. Further, the model was generalised to include the probability P_0 of having a leading π^0 in a given collision, with an energy $(1-K)E$ lost into the electromagnetic branch in that collision. The obtained number of muons as a function of the inelasticity K (for $E = 10^{18}$ eV) is shown in Fig. 1, for different values of P_0. As observed, in this toy model the N_μ slope depends critically on the nature of the leading particle: changing the probability P_0 of a leading neutral pion from 0 to 1/3 inverts this slope. If the inelasticity is relatively low, even a moderate fraction of leading neutral pions may cause a decrease of the number of produced muons. Thus, one easily moves from the behaviour predicted in the Matthews-Heitler model (section III-C) to the first hadronic collisions model (section III-B, equation (8)). It should however be noted that this simplified approach cannot correctly reproduce the complexity of the shower development. More detailed simulations will be presented below, and used both to prove the validity of the argument and to derive our results.
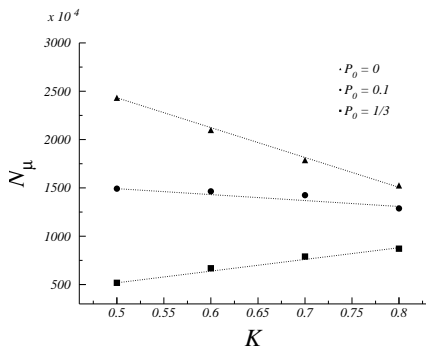

Fig. 1. The toy model prediction for the number of muons as a function of the inelasticity is shown, for different values of the probability P_0 of having a leading π^0 and $E = 10^{18}$ eV. The lines are only to guide the eye.

E. Hybrid shower simulations

In order to validate the argument and derive predictions for the behaviour of X_{max} and N_μ as a function of the energy, we need a shower simulation tool which is detailed enough to produce reliable results, and fast and flexible enough to allow the $ad - hoc$ introduction of the predictions of the percolation model that will be used. The hybrid, one dimensional simulation method described in [22] was used. It is a fast, one dimensional calculation, which provides predictions for the total number of charged particles and muons above several energy thresholds along the shower axis, as well as for the fluctuations of the electromagnetic and hadronic components of the shower. The method is based on precalculated showers (sub-threshold particles are treated with a library of profiles based on pre-simulated pion-initiated showers) and a bootstrap procedure to extend the shower library to high energy. The

SIBYLL hadron interactions model was used. The results have been cross-checked with QGSJET. In Fig. 2, the number of muons ($E > 1$ GeV) as a function of K is shown, for $E = 10^{18}$ eV, and for different values of the fraction of leading π^0's, P_0. This confirms the indication obtained above with the toy model: The slope of N_μ vs. K depends strongly on P_0. Studies performed with both QGSJET and SIBYLL show that
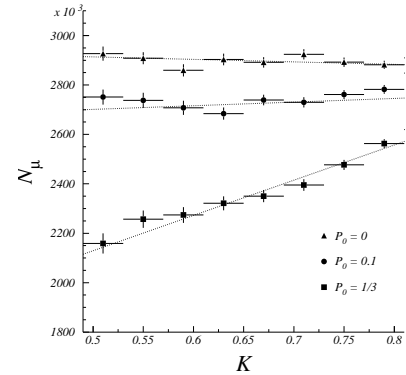

Fig. 2. The hybrid simulation prediction for the the number of muons ($E > 1$ GeV) as a function of the inelasticity is shown, for different values of the probability P_0 of having a leading π^0 and $E = 10^{18}$ eV. The lines are only to guide the eye.

the prediction of CORSIKA for N_μ vs. K is essentially flat. In QGSJET, the fraction of leading π^0's in the first hadronic collision goes roughly from 10% to 15% as the primary energy goes from 10^{15} eV to 10^{20} eV, but their spectrum is relatively soft. The variation of P_0 in SIBYLL is very small (of the order of 1% for the same energy range).

The combined effect of K and P_0 has thus an effect on muon production, and this is particularly true if the inelasticity has relatively low values. We now need a consistent model with predictions for the values of these variables.

F. String percolation model

Essentially, all existing high energy strong interaction models based on QCD, and QCD evolution, predict an increase with energy – not a decrease – of the inelasticity K [23]. The same is true for the hadronic generators SIBYLL [24] and QGSJET [25], used in cosmic ray cascade analysis. This happens because evolution in the energy implies transfer of energy from valence partons or strings, or from bare Pomeron diagrams, to sea partons or strings, or to multi-Pomeron contributions. However, in models with percolation of partons or strings, one expects the inelasticity K to decrease with energy above the percolation threshold [10]. In the framework of the Dual String Model [26] – but we believe the argument is more general – what happens at low energy is the transfer of energy from the valence strings to sea strings (and K increases), while at higher energy the strings start to overlap and a cumulative effect occurs: the length in rapidity of fused (percolated) strings is larger [27]. At some stage, close to percolation threshold, the percolating strings take over the

valence strings, and from then on K decreases with the energy. Percolation is, in fact, a mechanism for generating fast leading particles. In Fig. 3 we show the energy dependence of K in the
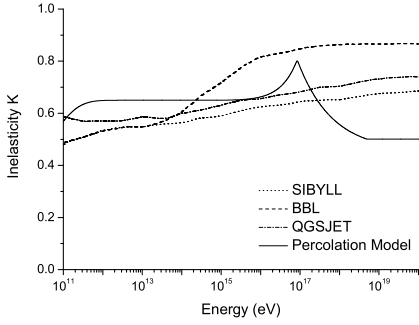

Fig. 3. Energy dependence of the average inelasticity K. Conventional QCD model results are from [28]: dotted curve is SIBYLL simulation; dashed-dotted curve is QGSJET simulations; dashed is a model of [28]; full line is our percolation model.

case of our string percolation model [10], in comparison with K determined from QCD inspired models, without percolation (see for instance [28]). As discussed in [10], this model was tuned taking into account multiplicity data from accelerator energies and places the percolation threshold at $E \simeq 10^{17}$ eV. In the reference, predictions for the inelasticity and multiplicity behaviour were made and their effect on X_{max} data was discussed. Above the percolation threshold the inverted slope of K vs. E is clearly visible. In the relevant energy range, K varies roughly from 0.8 to 0.55. Percolation will also affect the particle multiplicity, and we follow the treatment in [10], where we see that this effect can be introduced through the colour summation reduction factor $F(\eta)$, where η is the transverse density. Let us now turn to the evolution with energy of the probability P_0 of having a non-baryonic primary. As energy increases, the number of sea strings increases. These are, however, low energy strings, and, without percolation, the initial proton remains dominant. When sea strings percolate, larger "sea clusters" are formed, and other particles can be produced. It is a prediction of our percolation model that above the percolation threshold, because sea strings are of the type quark-antiquark, the probability of having a leading π^0 will tend to 1/3.

IV. RESULTS

We now have a string percolation model which predicts modifications on the variables K, P_0 and $< n >$ characterising hadronic collisions above a percolation threshold. Inserting these predictions into the hybrid shower simulation tool described in section III-E, results for the behaviour of N_μ and X_{max} as a function of the energy can now be derived and compared with the available data (see section II). The percolation prediction for the inelasticity K was introduced in the hybrid simulations for hadronic collisions above 10^{16} eV.

Above the percolation threshold ($E \sim 10^{17}$ eV), the probability of having a leading π^0 of P_0 was set to 1/3 and the percolation multiplicity reduction factor $F(\eta)$ was introduced. In Fig. 4(left) our results for N_μ vs. E are shown and compared with Akeno/AGASA data [12], [29]. A value of $\beta = 0.83$ is obtained from simulation, while without percolation we had $\beta = 0.89$. This is in very good agreement with the measured value of $\beta = 0.84 \pm 0.02$. This result was obtained with SIBYLL. A cross-check with QGSJET shows that the introduction of percolation gives a similar decrease, with β going from 0.92 to 0.86. The percolation model thus gives the right tendency, with a slight dependence with the underlying hadronic model. In this study a primary proton was considered. The result is however more general, as it has been shown in [6] that, for fixed composition, the value of β is essentially the same for proton and for Fe. The curve shown in the figure corresponds to shifting our proton result, using equation (2), to an intermediate and constant composition $A \sim 20$. This is obviously approximate, meant to show that data can be accomodated with a constant composition within this mode, and to give a rough indication of the effective mass number. The result of $\beta = 0.73$ quoted by HiRes-Mia can hardly be accommodated in this model, as it would imply values of P_0 going well beyond the expectations. It is interesting to note that at very high energies ($E \gtrsim 10^{19}$ eV) the π^0's starting to participate in the shower (i.e. to interact) and hence contribute to the production of muons. This causes an increase in the slope. A slight effect is already seen in our simulations. Finally, as muon data is often presented as muon density $\rho_\mu(600)$ vs. E, it is worth noting that CORSIKA [14] was used to check that $\rho_\mu(600)$ behaves much like N_μ as a function of the energy (with N_μ slightly steeper, but differences of the order of 0.02). In Fig. 4(right) the results obtained for X_{max} vs. E are shown and compared with existing data. Again, a primary proton has been considered, and the results have been shifted to an intermediate and constant composition using equation (1). It should be noted that this treatment is both approximate and model-dependent. It allows us to say that, roughly, our percolation model combined with SIBYLL or QGSJET can reproduce the data for a constant and medium-light (15-25) average mass number. We see that the obtained curve is reasonably consistent with data and with the result presented in [10] using a simple model similar to the one described in section III-B. A slight overshooting tendency is observed above $10^{18.5}$ eV. Our percolation model with a medium constant composition follows the tendency seen in the X_{max} data and usually interpreted as the composition going from heavy to light. However, it is not able to reproduce the apparent change in the elongation rate above $10^{18.5}$ eV. More data in this energy range confirming this feature would be most welcome.

V. SUMMARY AND CONCLUSIONS

In this paper we have shown that, in the interpretation of the energy dependence of the depth of the shower maximum and of the muon content in high energy cosmic ray showers
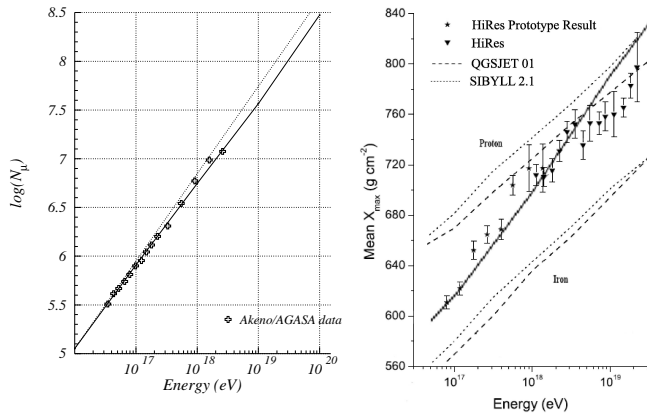
Fig. 4. (left) The number of muons as a function of the primary energy. The results of our model (full line) are shown and compared with Akeno/AGASA data (symbols). The dotted line shows $\beta = 0.9$, in agreement with current standard models; (right) The depth of the shower maximum as a function of the primary energy. The results of our model (full line) are shown and compared with HiRes data (symbols). SIBYLL and QGSJET predictions for proton and Fe initiated showers are shown for comparison [18].

($E \gtrsim 10^{17}$ eV), other variables besides the composition may play an important role, in particular those characterising the first (high energy) hadronic collisions. The inelasticity K, the nature of the leading particle P_0, and the particle multiplicity $< n >$ were discussed. We developed a model which includes a hybrid one-dimensional Monte Carlo simulation [22] and expectations from the percolation scenario [10]. For the first time the predictions of a percolation model for the different variables were included systematically and used to derive predictions on the behaviour of the depth of the shower maximum X_{max} and on the number of muons in the shower N_μ as a function of the energy. With the present model, we were able to explain the data in a reasonably consistent way, fairly describing the trends seen in data. It should however be noted that the development of a detailed Monte Carlo calculation is now required for a better understanding of data at all energies. In fact, in the present approach, percolation has been introduced 'ad-hoc' in the hybrid simulation. More complex effects may arise when it is fully introduced within the hadronic interactions model. Moreover, the model (in particular the behaviour of the number of strings as a function of the energy [10]) is tuned for high energy, the region where the percolation threshold arises ($E \gtrsim 10^{17}$ eV). The next step will thus be to create a full Monte Carlo simulation including percolation, adapted to the low and high energy regions, and also allowing to include composition in a proper way.

ACKNOWLEDGEMENTS

REFERENCES

[1] T. Shanta, Nucl. Phys. B 75A (1999) 27; K. Asakamori et al., Ap. J. 502 (1998) 278; M.L. Cherry et al, Proc. 26th Int. Cosmic Ray Conference, Salt Lake City, 3 (1999) 163.
[2] J. Linsley and L. Scarsi, Phys. Rev. Lett. 9 (1962) 123.
[3] S. Fukui et al., Progr. Theor. Phys. (Suppl.) N6 (1960) 1.
[4] J. Linsley, Proc. 18th International Cosmic Ray Conference, Bangalore 12 (1983) 135; Watson and Walker, J. Phys. G7 (1981) 1297; H.E. Bergeson, J.W. Elbert et al., 16th International Cosmic Ray Conference, Kyoto 8 (1979) 166 and 167; R.M. Baltrusaitis et al., Nucl. Instrum. and Methods A240 (1985) 410.
[5] M. Nagano and A.A. Watson, Rev. Mod. Phys. 72 (2000) 689, and references therein.
[6] M. Nagano, D. Heck, K. Shinozaki, N. Inoue, J. Knapp, Astrop.Phys.13 (2000) 277.
[7] J. Alvarez-Muñiz et al., 'Percolation and high enegy cosmic rays above 10^{17} eV", hep-ph/0608050, submitted to Astrop. Phys.
[8] R. Abbassi et al., Ap. J. 622 (2005) 910.
[9] L.W. Jones, Nucl. Phys. B75A (1999) 54; A. S. Chou, astro-ph/0606742
[10] J. Dias de Deus, M.C. Espirito Santo, M. Pimenta and C. Pajares, Phys.Rev.Lett.96 (2006) 162001.
[11] T. Antoni et al.,Astrop. Phys. 14 (2001) 245; W.D. Apel et al.,Astrop. Phys. 24 (2006) 467.
[12] N. Chiba et al., Nucl. Instrum. and Methods A311 (1992) 338; N. Hayashida et al., J. Phys. G 21 (1995) 1101; N. Hayashida et al., Proc. 25th ICRC (Durban) 4 (1997) 145.
[13] T. Abu-Zayyad et al. [HIRES-MIA Collaboration], arXiv:astro-ph/9911144. T. Abu-Zayyad et al. [HIRES-MIA Collaboration], Astrophys.J.557 (2001) 686.
[14] J. Knapp et al, Nucl. Phys. B (Proc. Suppl.) 52B (1997) 136; O. Heck et al., Nucl. Phys. B (Proc. Suppl.) 52B 139.
[15] B.R. Dawson, R. Meyhandan, K. M. Simpson, Astrop.Phys.9 (1998) 331.
[16] M. Heitler, 'Quantum Theory of Radiation", Oxford University Press (1944).
[17] J. Linsley, Proc 16th International Cosmic Ray Conference, Kyoto 9 (1979) 274.
[18] L. Anchordoqui, M. T. Dova, A. Mariazzi, T. McCauley, S. Paul, S. Reucroft, J. Swain, Annals Phys.314 (2004) 145.
[19] T.K. Gaisser, R.J. Protheroe, K.E. Turver, T.J.L. Mccomb, Rev.Mod.Phys.50 (1978) 859.
[20] J. Matthews, Astrop.Phys.22 (2005) 387.
[21] T. K. Gaisser, 'Cosmic rays and particle physics", Cambridge University Press, Cambridge (1990).
[22] J. Alvarez-Muñiz, R. Engel, T.K. Gaisser, J. A. Ortiz, T. Stanev Phys. Rev. D 66 (2002) 033011.
[23] B.Z. Kopeliovich, N.N. Nikolaev and I.K. Potashnikova, Phys. Rev. D39 (1989) 769; A. Capella, U. Sukhatme, C.I. Tan and J. Tran Thanh Van, Phys. Reports 236 (1994) 225; J. Ranft, Phys. Rev. D51 (1995) 64; L. Durand and H. Pi, Phys. Rev. Lett. 58 (1987) 303; H.J. Drescher, A. Dumitru and M. Strikman, hep-ph/0408073; S. Ostapchenko, hep-ph/0501093; I.N. Mishustin and J.L. Kapusta, Phys. Rev. Lett. 88 (2002) 112501.
[24] R.S. Fletcher, T.K. Gaisser, P. Lipari and T. Stanev, Phys. Rev. D63 (2001) 054030.
[25] N.N. Kalmykov, S.S. Ostapchenko and A.I. Pavlov, Nucl. Phys. Proc. Suppl. 52B (1997) 17.
[26] N.S. Amelin, M.A. Braun and C. Pajares, Phys. Lett. B306 (1993) 312; N.S. Amelin, M.A. Braun and C. Pajares, Z. Phys. C63 (1994) 507; M.A. Braun and C. Pajares, Eur. Phys. J. C16 (2000) 349; M.A. Braun and C. Pajares, Phys. Rev. Lett. 85 (2000) 4864; C. Pajares, D. Sousa and R.A. Vásquez, Phys. Rev. Lett. 86 (2001) 1674.
[27] M.A. Braun, E.G. Ferreiro, F. del Moral and C. Pajares, Eur. Phys. J. C25 (2000) 249.
[28] H.J. Drescher, astro-ph/0411143.
[29] M. Takeda et al, Astrop. Phys. 19 (2003) 447.