Multiple scattering of the fluorescence light from EAS

Maria Giller and Andrzej Smialkowski
Division of Experimental Physics, University of Lodz
Email: mgiller@kfd2.phys.uni.lodz.pl

***Abstract*— One of the methods to study the highest energy cosmic rays is to observe the fluorescence light emitted sideways by the extensive air showers (EAS). To reconstruct a shower cascade curve, N(X), from the observations of the light arriving from the directions towards the subsequent shower track elements, it is necessary to take into account the multiple scatterings that photons undergo on its way from the shower to the detector. This effect is important, particularly for distant showers. In contrast to some Monte-Carlo treatments, we present here an analytical method to calculate the Rayleigh and Mie scatterings in a constant density atmosphere. Our method consists in treating separately the consequent 'generations' of the scattered light. The results can be scaled to various distances measured in the mean scattering length.**

I. Introduction

A distant shower can be treated as a point isotropic light source moving with light velocity c along the shower track. At the fluorescence detector camera, measuring the angular (and temporal) distribution of the arriving light, it causes an elongated track of hit pixels (PMT's), each having a small angular field of view. Usually it is assumed that it is only the light emitted in the pixel's field of view which arrives, after some attenuation, at the pixel. However, the attenuation consists in scattering the photons away of the field of view of the considered pixel.

The angular and time distributions of photons scattered in the Rayleigh process are presented in [1]. Here we continue by calculating what fraction of the scattered photons comes back to the pixel and what is the impact on observations of the extensive air showers when both molecular (Rayleigh) and aerosol (Mie) scattering take place.

II. The method

- The scattering processes are:on molecular (Rayleigh) and aerosol (Mie). The atmosphere is homogeneous (so far).
- Scattered light is a sum of photons scattered once n_1, twice n_2, and so on.
- We calculate analytically angular and time distributions of consequent generations n_i.
- We apply our calculations to the fluorescence detector camera in the Auger experiment (a pixel field of view is $1.5°$).

To determine the effect of the multiple scattering of the EAS fluorescence light we have to consider a moving, isotropic point source. Let $J(\theta, \phi; R, t)d\Omega dt$ be the number of photons arriving in the solid angle $d\Omega(\theta, \phi)$ in the time interval

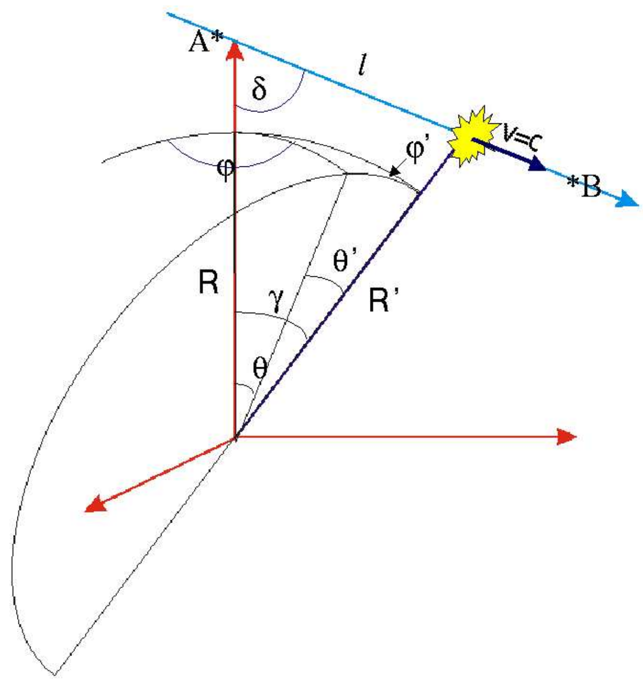

Fig. 1. Geometry for a moving point source of light. The observer (a pixel in the camera) is at the beginnig of the coordinate system and looks in direction $\theta = 0$.

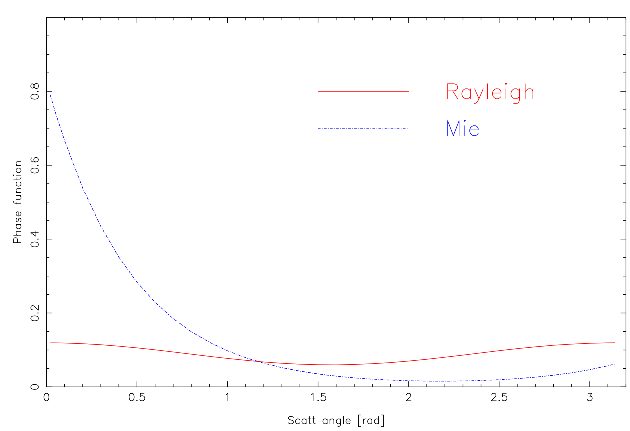

Fig. 2. Phase functions

$(t, t + dt)$ at the unit surface at distance R from the point on the shower axis determined by $\theta = 0$.
Then

$$J_i(\theta, \varphi, t; R) = \int_A^B j_i(\theta', t'; R') \cdot Cdl \tag{1}$$

where

$$j_i(\theta', t'; R') = \frac{1}{2\pi\, sin\theta' |cos\theta'|} \frac{d^2 n_i}{d\theta' dt'} \tag{2}$$

A and B are the limiting points of the shower track and Cdl is the number of photons produced along the track element dl (Fig.1).
For the first generation of scattered photons n_1 we have [1]:

$$\frac{d^2 n_1}{d\theta' dt'} = \frac{3}{16\pi} \frac{k \cdot c}{R'^3} e^{-k\tau} \cdot f(\alpha) \frac{cos^2 \frac{\beta' + \theta'}{2}}{sin\theta'} \tag{3}$$

where β' is deduced from:

$$tg\frac{\beta' + \theta'}{2} = \frac{\tau - cos\theta'}{sin\theta'} \tag{4}$$

$f(\alpha)$ is the phase function for Rayleigh or Mie scattering (Fig.2) [1], [2], [3].

Any next generations n_{i+1} can be calculated using the previous one n_i :

$$\frac{d^2 n_{i+1}}{d\theta' dt'} = \frac{3}{8\pi} \frac{k}{R'^2} \int_0^{\beta_{max}} e^{-x'k/R'} \cdot r^2 d\beta \cdot \\ \cdot \int_0^{\pi} \frac{dn_i(r, \theta_i, t' - x'/c)}{d\theta_i dt_i} d\theta_i \int_0^{\pi} \left[f(\alpha) \right] d\phi' \tag{5}$$

where $t' = t - l/c; t = ct'/R'$.
If the angle between the shower axis and the direction $\theta = 0$ is δ then

$$R' = R\sqrt{1 - \frac{2l}{R} cos\delta + \frac{d^2}{R^2}} \tag{6}$$

where l is the distance of the element dl from the point on the shower with $\theta = 0$. We also have that $t' = t - l/c$ and $cos\theta' = cos\gamma\, cos\theta - sin\gamma\, sin\theta\, cos\varphi$, where γ is determined from: $l/R = sin\gamma / sin(\gamma + \delta)$ (Fig.1).

To calculate how many scattered photons arrive at a particular pixel of the detector camera, one has to integrate $J(\theta, \phi; R, t)$ over its field of view.

Results depend on :

- $k = R/\lambda$ (distance in units of the total mean scattering length).
- time in units of R/c.
- δ - angle between shower track and line of sight of the camera.

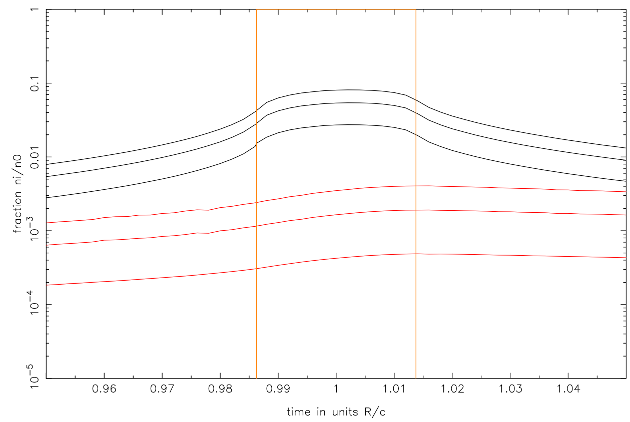

Fig. 3. Time distributions (in units of R/c) of once (black lines) and twice (red lines) scattered light collected by $1.5°$ camera pixel as a fraction to the direct light. Shower track is perpendicular to the line of sight. Yellow rectangle describes the unscattered component n_0. Curves are for distances $k = 1, 2, 3$ from top to bottom.

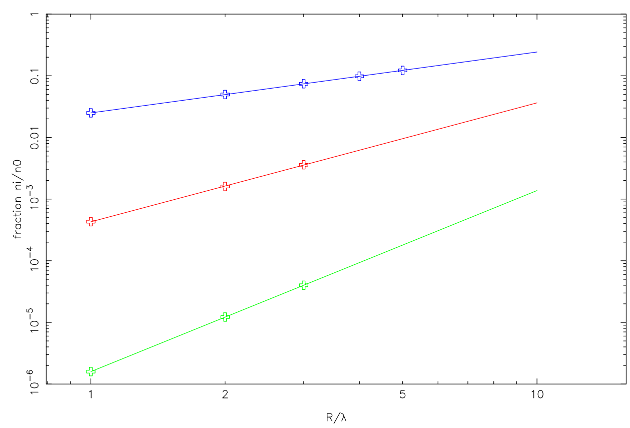

Fig. 4. Fraction of the scattered light (one, two and three times) collected by $1.5°$ camera pixel as a function of $k = R/\lambda$, in time window specified by the direct light arrival. Shower track perpendicular to the line of sight. The lines are proportional to k^i.

III. RESULTS

Figure 3 shows the time distributions of the scattered photons (ones and twice) collected by a single $1.5°$ camera pixel for three distances $R/\lambda = k = 1, 2, 3$. The scattered light is presented as a fraction of the direct (unscattered) light. The track of the constant light source is perpendicular to the line of sight ($\delta = 90°$).

Next, we integrate the number of scattered photons in the time window when direct photons arrive from the shower track element just seen by our pixel (this is the time interval limited by vertical lines). In Figure 4 we present the dependence of the scattered light fraction on the distance R/λ. All generations of the scattered photons have general behaviour: the fraction of the scattered light scales as $k^i = (R/\lambda)^i$ for the $i - th$ generation. It can be seen that

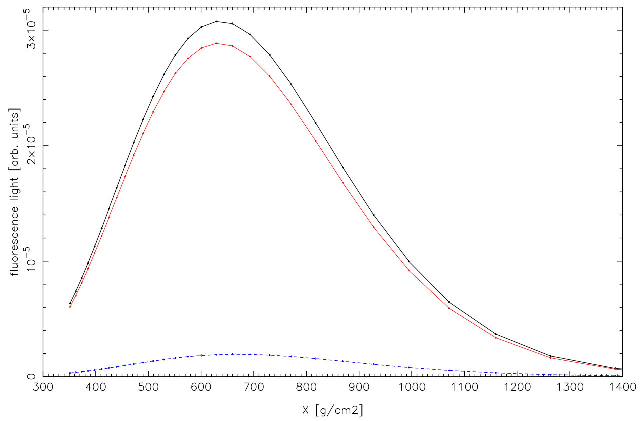

Fig. 5. Effect of multiple scattering on fluorescence extensive air shower profile. The shower with $X_{max} = 750 g/cm^2$ (which corresponds to proton 10^{19} eV), zenith angle $60°$ and core position distant at k=4, shower detector plane perpendicular to the horizon. The upper black curve is total light, the red curve is direct light flux when multiple scattering photons (blue curve) are subtructed

the first generation dominates within distances of detection interest. It should be noted that this fraction also depends on δ [2].

Figure 5 shows an example illustrating how multiple scattering of the fluorescence photons changes the reconstruction of the detected light profile of an air shower. Here we apply a longitudinal profile (Gaiser-Hillas) of fluorescence photons produced along extensive air shower track with the maximum at $X_{max} = 750 g/cm^2$ (corresponding to a primary proton with energy 10^{19} eV), inclined by zenith angle $60°$ towards the detector and the core position at $k = 4$. The upper black curve represents light collected by the camera, the lower red curve represents direct light flux when multiple scattering effect is taken into account.

We have evaluated that without including multiple scattering, the number of particles and thus the total energy of this particular shower would be overestimated by $\sim$ 5%. This overestimation could be as large as 10% for distant showers with highest energies observed. Number of scattered photons depends not only on distance to the shower but on geometry of the shower as well. (angle between the line of sight of each pixel and the shower).

IV. Conclusions

- It is possible to treat the multiple scattering of the fluorescence light analytically, with some numerical help.
- The fraction of the scattered light scales as $k^i = (R/\lambda)^i$ for the $i - th$ generation, and time distributions depends on R/c.
- The first generation of the scattered photons dominates.
- The effect of the multiple scattering is non negligible for distant showers (distance $>$ 20 km). When taken into account it can change the primary energy evaluation down by $\sim 10\%$.
- Results were compared with Monte Carlo work [3] and show good agreement.
- The inhomogeneous atmosphere work is in progress.

Acknowledgment

This work is supported by Polish Ministry of Science and Higher Education; grant No 1P03D01430.

References

[1] Giller M., Smialkowski A. ,2005, Proceedings of the 29th International Cosmic Rays Conference, Pune, India
[2] Giller M., Smiakowski A., 2006, (in preparation)
[3] Roberts M. D., 2005, J. Phys. G: Nucl. Phys. 31, 1291