The 210Pb bremsstrahlung component in the background spectrum of lead shielded γ-spectrometers

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Abstract- **The expected interactions of weakly interacting massive particles with the matter are by the nuclear recoils so the deposited energies in detector are up 100 keV. The bremsstrahlung of ^{210}Pb contributes to this spectral region. The 46.5 keV γ-intensity of ^{210}Pb in the background can be easily reduced by inner lining, but the bremsstrahlung from the 1.2 MeV maximal energy β-decay will reach the lead shielded Ge detector. The spectrum of this bremsstrahlung is calculated by numerically fitting the β-spectrum and integrating the Koch-Motz formula. The absorption of the bremsstrahlung spectrum in the lead and detection efficiencies for Ge detector are calculated by the effective solid angle algorithm, using correction for photopeak/compton ratio of cross sections in Ge. By comparison with the measured spectrum, it is shown, that for the lead with 25 Bq/kg of ^{210}Pb, up to 500 keV, the bremsstrahlung contribution to the background is about 20%. The calculated bremsstrahlung intensity is in good agreement with the published experimental results.**

I. Introduction

CONTEMPORARY theories of dark matter predict that weakly interacting massive particles (WIMP) interact with matter by nuclear recoils leaving in the detector energies up 100 keV. The more massive the WIMP, the larger deposited energy, but also the smaller the event rate since the number density of WIMPs decreases linearly as their mass increases. Most WIMP experiments are based on lead shielded detectors, thus the bremsstrahlung of ^{210}Pb contributes to the expected WIMP spectral region.

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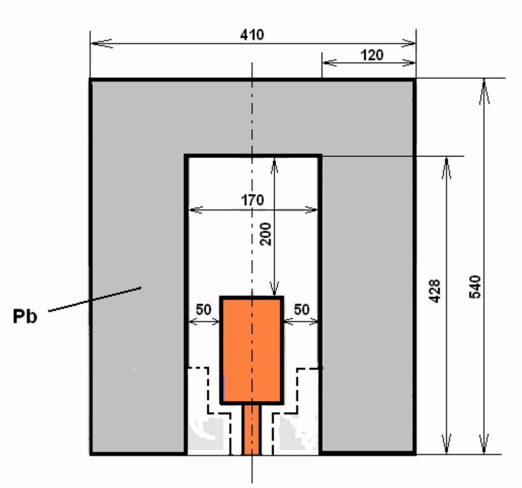

FIG.1. Lead shield of HPGe detector

Actually, Pb-210 emits only a weak (4.05%) low energy γ-ray at 46.5 keV, while its daughter Bi-210, is an almost pure β^- emitter ($E_{\beta max}$=1.16 MeV). The contribution of this bremsstrahlung to the background of lead shielded Ge detectors was experimentally studied by Heusser [1] without an attempt for quantitative explanation.

In the present paper we describe a semi empirical method, for the estimation of bremsstrahlung intensity, induced by Pb-210, in the background of the GMX type "ORTEC" HPGe spectrometer with nominal efficiency of 32% (V=160 cm^3 volume). The ^{210}Pb content of the lead shield is measured [2] to be A_s=25±5 Bq/kg. The shield has cylindrical shape with outer diameter of Φ=410 mm, and wall thickness of 120 mm. Schematic view of the detector position inside shield is shown on Fig. 1.

II. Calculation of the Emission Spectrum and Detection Efficiencies

A. The Probability of Electron Emission from β Decay of Bi-210

Using the experimental β spectrum [3], analytical function of energy distribution of emitted Bi-210 electrons $N(E_\beta)$, i.e. emission probability per unit energy interval, is found. Computer program "Table Curve 2" is used for fitting, and result (Fig.2) can be expressed in following form:

$$N_\beta(E_\beta) = a + b\,E_\beta + c\,E_\beta^{3/2} + d \cdot E_\beta^{\;2} \cdot \ln E_\beta + g\,e^{-E_\beta} \quad (1)$$

where a= - 1074.2113, b= 1307.8285, c= - 631.69764, d= 31.006627, g= 1084.6598. E_β is kinetic energy of electron in dimensionless units, relative to 1 MeV.

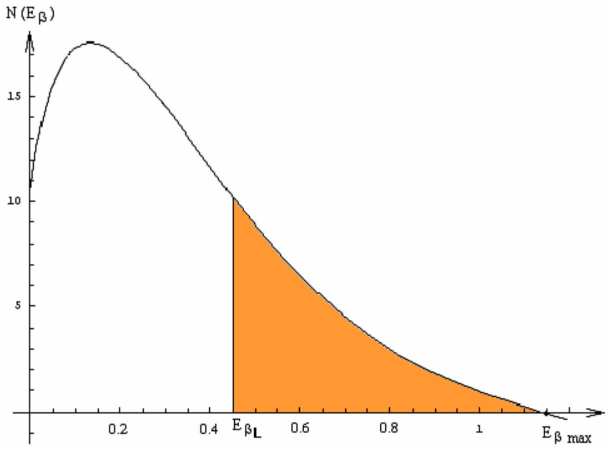

FIG. 2. Fitted energy distribution of electrons from Bi-210 β-decay. E_β- kinetic energy of electrons

The probability of electron emission $w(E_{\beta L}, E_{\beta max})$ in the energy interval ($E_{\beta L}$,$E_{\beta max}$=1.14) is calculated from:

$$w(E_{\beta L}, E_{\beta\max}) = k \int_{E_{\beta L}}^{E_{\beta\max}} N(E_\beta)\,dE_\beta \quad (2)$$

and it corresponds to marked area below the curve on Fig.2.

The normalization factor k=0.107787 satisfies condition

$$k \int_{0}^{E_{\beta\max}} N(E_\beta)\,dE_\beta = 1 \quad (3)$$

Lower integration limit $E_{\beta L}$ was varied from 0.05 to 1.10 with 0.05 steps.

The spectrum of emitted electrons is different from spectrum of electrons which contribute to bremsstrahlung emission, because one part of energy electrons loses in collisions.

Also, contribution to emission of bremsstrahlung photons at energy E_γ gives only electrons that have energy higher than E_γ: $E_\gamma \le E_\beta \le E_{\beta max.}$

B. The Probability of Bremsstrahlung Emission

The probability of bremsstrahlung emission $d\sigma/dE_\gamma$[cm^2/MeV] depends of initial kinetic energy of electron E_β and energy of emitted photon E_γ [4]:

$$\frac{d\sigma}{dE_\gamma} = f(E_\beta, E_\gamma) \quad (4)$$

We considered the bremsstrahlung emission inside lead (Z=82). In that case, the explicit form can be written as

$$\frac{d\sigma}{dE_\gamma} = \frac{1.558947x10^{-23}}{E_\gamma} \times \left\{ (1+\varepsilon^2)\left[\frac{\phi_1(\xi)}{4} - 1.800665\right] - \frac{2}{3}\varepsilon\left[\frac{\phi_2(\xi)}{4} - 1.800665\right]\right\} \quad (5),$$

with

$$\varepsilon = \frac{E_\beta - E_\gamma + 0.511}{E_\beta + 0.511}, \quad (6)$$

and screeneng functions

$$\phi_1(\xi) = 20.863 - 2\ln\left[1 + (0.55846\,\xi)^2\right] - 4\left(1 - 0.6\,e^{-0.9\xi} - 0.4\,e^{-1.5\xi}\right) \quad (7)$$

$$\phi_2(\xi) = \phi_1(\xi) - \frac{2}{3}\left(1 + 6.5\xi + 6\xi^2\right)^{-1} \quad (8)$$

depending on the parameter

$$\xi = \frac{0.511 E_\gamma}{(E_\beta + 0.511)(E_\beta - E_\gamma + 0.511)4.34448}. \quad (9)$$

C. The Emission Spectrum of Bremsstrahlung

The number of emitted bremsstrahlung photons in lead shield, per unit of energy and unit of time is obtained by the following formula

$$\frac{dR(E_\gamma)}{dE_\gamma} = A \cdot w(E_{\beta L}, E_{\beta\max})\,c(E_{\beta L}) \times \int_{E_{\beta L}=E_\gamma}^{E_{\beta\max}} \rho_{Pb}\frac{N_A}{M_{Pb}}\bar{r}_\beta(E_\beta)\frac{d\sigma}{dE_\gamma}(E_\beta, E_\gamma)\,dE_\beta \quad \left[\frac{1}{sMeV}\right] \quad (10),$$

where: A is the total ^{210}Pb activity in the shielding material (A=$A_S \cdot m$=25Bq/kg ·708kg=17700Bq), w($E_{\beta L}$, $E_{\beta max}$) is the probability of electron emission in the energy interval ($E_{\beta L}$, $E_{\beta max}$), $E_{\beta L}$ is the lowest β electron energy contributing to the probability of bremstrahlung emission with energy E_γ ($E_{\beta L} = E_\gamma$) and c($E_{\beta L}$) is the function that takes into account the collision-ionization losses in lead. It is simply the $\left(\frac{dE_{\beta L}}{dx}\right)_{rad} / \left(\frac{dE_{\beta L}}{dx}\right)_{coll}$ ratio dependence [5] on the electron energy.

The integral term represents the total probability of bremstrahlung emission on energy E_γ generated by all electrons with energy higher than E_γ. The first factor under the integral $\rho_{Pb}\frac{N_A}{M_{Pb}}$ determines the number of lead atoms per unit of volume (ρ_{Pb} is density, N_A is Avogadro's number, M_{Pb} is atomic weight).

The function

$$\bar{r}_\beta(E_\beta) = \frac{1}{\rho_{Pb}} 0.412 \times E_\beta^{(1.265 - 0.0594\ln E_\beta)} \quad [cm] \quad (11)$$

describes the mean range of the electrons with energy E_β (in MeV) [6]. The last factor $\frac{d\sigma}{dE_\gamma}$ is the standard Koch-Motz probability for bremsstrahlung emission [4].

From (10) the finite spectral intensity $\frac{\Delta R(E_\gamma)}{\Delta E_\gamma}$ is derived by varying $E_{\beta L}$ in bins of 50 keV.

D. The Analysis of Self-absorption Effect and Detection Efficiencies of Bremsstrahlung Spectrum

The emission spectrum is significantly modified by self absorption in the thick wall of lead shield. The self absorption effect and the detection efficiencies of HPGe detector were calculated by the "ANGLE" computer code [7], [8]. Having in mind the cylindrical shape of shield, it was assumed that shield represents a photon source with Marinelli geometry in Pb matrix. Using equation:

$$\varepsilon_{Pb} = \frac{\Omega_{Pb}}{\Omega_{ref}} \varepsilon_{ref} \,, \tag{12}$$

the full peak detector efficiencies of photons from lead shield in energy range 50 keV-1.10 MeV were found. The calculated quantities Ω_{Pb} and Ω_{ref} are effective solid angles for Marinelli geometry and for reference cylindrical source respectively, while ε_{ref} is the measured total absorption peak efficiency for the reference source. The total absorption peak efficiency ε_{Pb} was corrected for the contribution of Compton events to the detector spectrum by multiplying it with the ratio $(\sigma_{Ph}+\sigma_C)/\sigma_{Ph}$, where σ_{Ph} is photo peak cross section and σ_C Compton scattering cross section in Ge, depending on photon energy [9].

TABLE I
THE EXPECTED BREMSSTRAHLUNG INTENSITIES IN SPECTRUM OF HPGE DETECTOR (''DETECTED'' BREMSSTRAHLUNG INTENSITIES)

E_γ [MeV]	$\frac{\Delta R(E_\gamma)_{DET}}{\Delta E_\gamma} \left[\frac{1}{s \cdot chn}\right]$
0.05	1.537×10^{-4}
0.1	1.738×10^{-4}
0.15	1.741×10^{-4}
0.2	1.669×10^{-4}
0.25	1.525×10^{-4}
0.3	1.461×10^{-4}
0.35	1.248×10^{-4}
0.40	1.078×10^{-4}
0.45	9.04×10^{-5}
0.5	7.41×10^{-5}
0.55	5.86×10^{-5}
0.60	4.72×10^{-5}
0.65	3.39×10^{-5}
0.70	2.39×10^{-5}
0.75	1.62×10^{-5}
0.80	1.06×10^{-5}
0.85	6.98×10^{-6}
0.90	4.14×10^{-6}
0.95	2.25×10^{-6}
1.00	1.02×10^{-6}
1.05	3.11×10^{-7}
1.10	3.45×10^{-8}

The "detected" bremsstrahlung intensities were derived from equation:

$$\frac{\Delta R(E_\gamma)_{DET}}{\Delta E_\gamma} = \frac{\Delta R(E_\gamma)}{\Delta E_\gamma} \cdot \varepsilon_{Pb} \cdot \frac{\sigma_{Ph} + \sigma_C}{\sigma_{ph}} \,. \tag{13}$$

The results are presented in Table I. The relative uncertainties of results were about 20 %, due to uncertainty of Pb-210 activity.

Because of our intent to compare the calculated bremsstrahlung spectrum with measured background spectrum, we expressed the bremsstrahlung intensities in units $\left[\frac{1}{s \cdot chn}\right]$ instead in $\left[\frac{1}{s \cdot MeV}\right]$, where 1chn =0.507 keV (2077 keV = 4093 chn).

III. RESULTS AND DISCUSSION

The "detected" spectrum, calculated for 22 points, is on Fig.3. compared with the measured background.

On Fig.4. the calculated bremsstrahlung spectrum for our detector (dots) is compared with one of the measured spectrum of Heusser [1] (upper spectrum) taken in the shallow underground laboratory. The same ^{210}Pb content in both leads (25 Bq/kg), the similar nominal detection efficiency of both detectors and similar ($\approx 4\pi$) geometry, makes this comparison meaningful. The agreement between the calculated and measured data on Fig. 4 is satisfactory. This means that the severe approximations made in the description of electron progression and bremsstrahlung emission are a posteriori fully justified.

The differences between calculated bremsstrahlung intensities for our detector and measured bremsstrahlung spectrum of Heusser might be caused due to:

- the mass difference of lead around detectors (our shield:~700 kg, experimental setup performed by Heusser:~100 kg).The attribution of the disagreement to the mass difference is supported by the increasing discrepancy for increasing photon energy (Fig.4). At higher photon energies the Pb transparency is higher and the mass influence is stronger.
- the difference of lead positions around detectors (in the experimental setup of Heusser, the lead was in contact geometry with detector)
- the difference between construction of the two detectors.

Due to the lack of data for Heusser's detector (the dimensions of Ge crystal and its position inside detector end-cap) our calculation could not be specified exactly his measurements.

The method described predicts the detected bremsstrahlung spectrum from the ^{210}Pb content of the lead shield, detector characteristics and shielding geometry. Thus these counts, unaffected by active vetoing techniques, can be safely subtracted from the measured data. In the low energy region (E_γ <100 keV) where the WIMP events are expected [10] our method can significantly improve the data recognition and filtering.

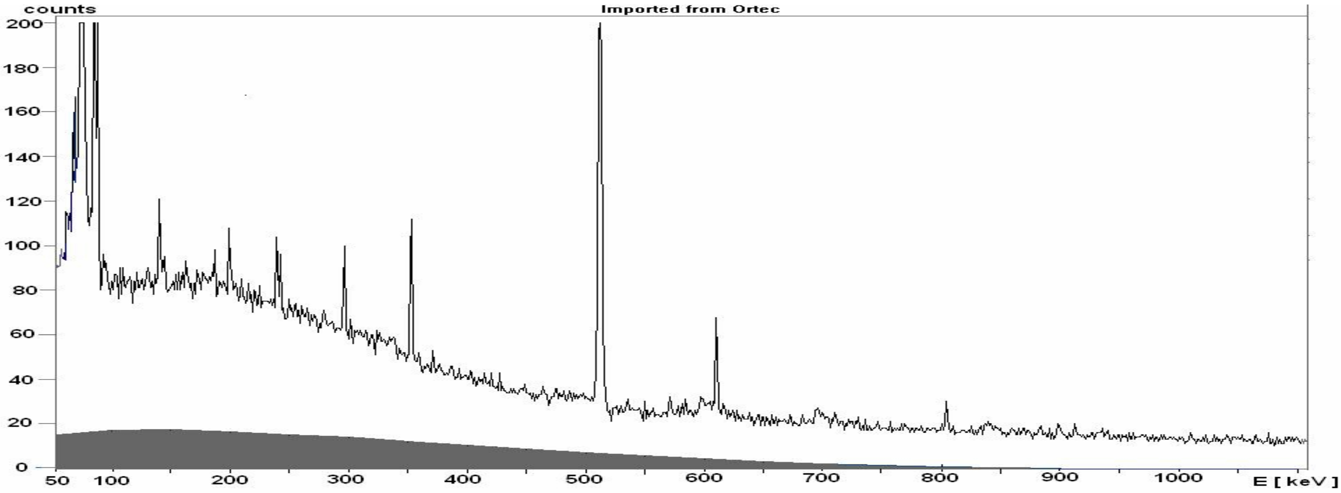

FIG.3. The "detected" bremsstrahlung spectrum (shadowed) compared with the measured background

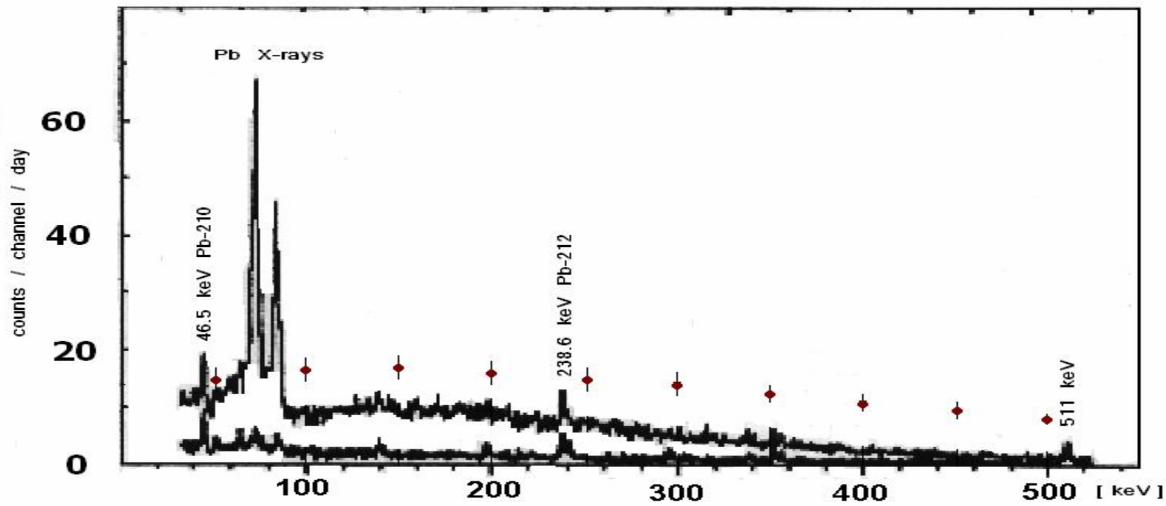

FIG. 4. The calculated bremsstrahlung spectrum for our detector (dots with error bars) compared with the measured spectrum of Heusser (upper spectrum)

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