A stochastic Montecarlo approach to Solar modulation of GCR: evaluation of the proton flux at several distances from the Sun

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***Abstract*— A 2D stochastic simulation model of GCR propagation in the heliosphere has been developed. We have tuned the model by using the measured parameters as solar wind speed and tilt angle both for solar positive and negative polarity periods. Drift effect is also included in the model. We have studied the solar modulation at several distancies from the sun and have evaluated the primary proton flux at the position of several planets.**

I. Introduction

Accurate models of heliospheric propagation of Galactic Cosmic Rays (GCR) need to take into account all the features of the solar cavity in order to reproduce the observed flux. We can mention for example the complex structure of the heliospheric magnetic field [1], after the measurements performed by the Ulysses satellite. We used the Parker field model for the heliosphere [2], that become time dependent due to the measurements connected to drift effects [3]. We have first produced a 1D heliospheric model using a stochastic simulation approach, (see [4] and [5]), useful for testing the method and successful in reproducing the main feature of the solar cavity. Then we have implemented a two dimensional (radius and heliolatitude) drift model of GCR propagation in the heliosphere [6] time dependent due to the variation of the measured values of the solar wind velocity in the ecliptic plane (V) and the tilt angle (α). This model is including curvature, gradient and current sheet drifts, which are depending on the charge sign of particles. In section III-A we present a comparison between the 1D and the 2D model. We then implemented in this model the possibility to reproduce the modulated flux at several distancies from the sun. In this way we obtain the primary CR flux at the position of the planets of the solar system.

II. Stochastic 2D Monte Carlo model

We have used the approach to study the Cosmic Rays propagation in the Heliosphere with a Monte Carlo technique. It is a stochastic Monte Carlo in two dimension (r and θ), and is based on the Fokker-Planck equation (hereafter FPE) for GCR transport in the heliosphere without drift terms [7] and [8]:

$$\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_{rr}\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(K_{\theta\theta}\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2 V\right)\frac{\partial}{\partial T}\left(\Gamma T f\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V f\right) \quad (1)$$

Where θ is the heliolatitude f is phase space distribution function, t is the time, T is the kinetic energy (per nucleon) r is the heliocentric radial distance, V the solar wind velocity and $\Gamma = (T + 2T_0)/(T + T_0)$ where T_0 is proton's rest energy. The first two terms in eq. 1 describe the diffusion of GCR in the heliosphere, the third term is adiabatic energy loss and the last one is convection by the outgoing solar wind. Thanks to a rigorous mathematical proof by Ito [9] there is an exact equivalence between the FPE and a set of stochastic differential equations (SDE), so we can apply this technique to our numerical simulation solving ordinary differential equations. If we call the coordinates variation in a small time step Δt for a test quasi-particle, the SDE equivalent to eq. 1 can be written as:

$$\begin{aligned}
\Delta r &= \frac{1}{r^2}\frac{d(r^2 K_{rr})}{dr} + V\Delta t + R_g\sqrt{2K_{rr}\Delta t} \\
\Delta\mu &= \frac{d}{d\mu}\left[\left(1-\mu^2\frac{K_{\theta\theta}}{r^2}\right)\right]\Delta t \\
&\quad + R_g\sqrt{2\left(1-\mu^2\right)\frac{k_{\theta\theta}}{r^2}\Delta t} \\
\Delta E &= -\frac{2VT\Gamma}{3r}\Delta t \qquad (2)
\end{aligned}$$

where $\mu = cos\theta$ and so $\Delta\mu = \Delta cos\theta$ is the latitudinal variation of the particle, V is solar wind velocity, R_g is the Gaussian distributed random number with unit variance, Δt is the time step of calculation. The radial diffusion coefficient is $K_{rr} = K_{||}cos^2\psi + K_{\perp}sin^2\psi$, where ψ is the angle between radial and magnetic field directions [7]. The latitudinal coefficient is $K_{\theta\theta} = K_{\perp}$. The parallel and the perpendicular diffusion coefficients are

$$K_{||} = K_0 \beta K_P(\Re) \frac{B_{\bigoplus}}{3B}$$
$$K_{\perp} = (K_{\perp})_0 K_{||} \tag{3}$$

where $K_0 = 1-6 \times 10^{22}$ cm^{2}s^{-1} [10], β is the particle velocity, $K_P(\Re)$ take into accounts the dependence on rigidity (in GV), $(K_{\perp})_0$ is the ratio between parallel and perpendicular diffusion coefficient, $B_{\bigoplus} \sim 5$ nT is the value of heliospheric magnetic field at the Earth orbit, and B is the Parker field [11]. We used the Parker model for the heliospheric magnetic field because this allows an analytical solution for drift velocities. In our model the solar wind speed $V(\theta)$ is a function of the heliolatitude θ [12]: $V = V_0(1 + sin\theta)$, for $0^o < \theta < 60^o$ and $V = 750$ kms^{-1}, for $60^o < \theta < 90^o$. V_0 is the velocity of solar wind in ecliptic plane. Drift effects are included through analytical effective drift velocities [13]. The average drift velocity is $v_d = \nabla \times (\frac{\beta\Re}{3B})$. $\Re$ is the CR particle's rigidity. The Parker model allows analytical solution for drift velocities and better evaluation of diffusion tensor. In this spiral field we added to the previous formulas (2), the three components of drift, the gradient, the curvature and drift along the neutral sheet to calculate the position of a test particle during a time step Δt:

$$\Delta r_d = \Delta r + (v_g + \overline{v_d^{ns}})\Delta t$$
$$\Delta\mu_d = cos\left[\arccos(\Delta\mu) + \arctan\left(\frac{v_\theta \Delta t}{r}\right)\right] \tag{4}$$

Δr_d is the radial variation with drift effect, $\Delta\mu_d$ is the latitudinal variation of the particle due to drift, v_g is the velocity of gradient drift, v_d^{ns} is the velocity of neutral sheet drift and v_θ is the velocity of curvature drift. As Local Interstellar Spectrum of protons (LIS) we used Burger's model [16]. Our model has been optimized by fine tuning the parameters $K_P(\Re)$ and $(K_{\perp})_0$ of the diffusion tensor, performing long time consuming simulations and comparing results with experimental data (flux spectrum at 1AU).

III. Study of the Modulation parameters

A. 1D vs. 2D model

We present here a comparison between the results of our previous 1D model (only radius as spatial contribution) and our new 2D model. With a fixed value of the diffusion coefficient $K_0 = 0.89 \times 10^{-7}$ au^{2}s^{-1} (corresponding to $K_0 \simeq 2.01 \times 10^{22}$ cm^{2}s^{-1}) the simple 1D model produces a flux much lower than 2D (the modulation strenght is higher, see Fig. 1). This may be due, in addition to the latitudinal variation included in the 2D model (see sec II) to the drift componets (gradient and curvature). The neutral sheet drift components effect is evaluated in sec. III-D

In particular diffusion coefficient for the 1998 period related to AMS-01 data (see sec.IV-A) can be estimated for the 1D

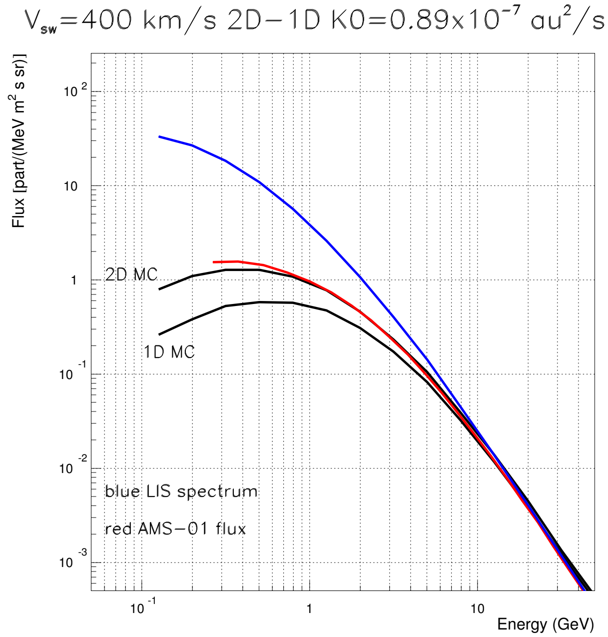

Fig. 1. Comparison of 1D and 2D model with AMS-01 data: K_0 and V_{sw} are fixed

model as $K_0 = 1.61 \times 10^{-7}$ au^{2}s^{-1}, corresponding to a modulation strenght (in agreement with independent estimations, see [18] and [19]) $\phi \simeq 510 - 550$ MV).

B. Diffusion coefficient

We evaluated the dependence of the modulation effect from the diffusion coefficient value, in the quasi linear theory approximation (i.e. $K_P = K_0\beta\Re$).

In the 2D version the analysis of the relation between the diffusion coefficient and the modulation strenght becomes more complex. The modulation as a function of the diffusion coefficient follows the expected behaviour, so higher values of K_0 corresponds to a lower modulation and vice versa (see Fig. 2 for CR spectra at Earth without NS drift and Table I -left part- related to CR spectra estimated at Pluto distance).

We evaluated also the dependence of our model on the ratio between parallel and perpendicular diffusion coefficient, over the commonly used range, $(K_{\perp})_0 = 0.01 - 0.05$. The modulation effect on the LIS is higher for lower values of $(K_{\perp})_0$ (these results are not shown here, just the best value for $A > 0$ period is used, as reported in IV-A).

C. Tilt angle α and Solar Wind velocity V_{sw}

For positive periods we have evaluated the effect on the model by changing the solar wind speed in the range $V_0 = 100 - 1000$ kms^{-1} and tilt angle in the range $\alpha = 10 - 50^o$. We used (see for complete description [14]) the tilt angle α as main parameter for the level of the solar activity: the higher the value of α the lower the expected GCR flux, for both solar field polarity. Besides for the same value of α a higher flux of protons is expected for $A > 0$. We also evaluated the effect

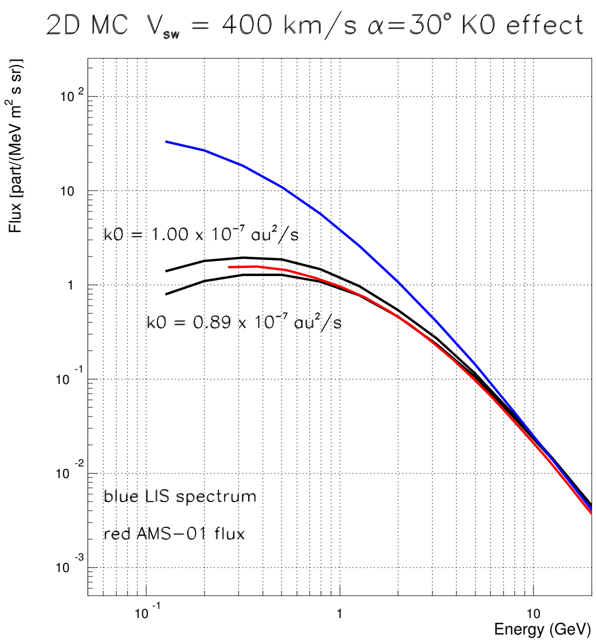

Fig. 2. 2D model: diffusion coefficient effect on the modulated CR flux at Earth, no NS drift

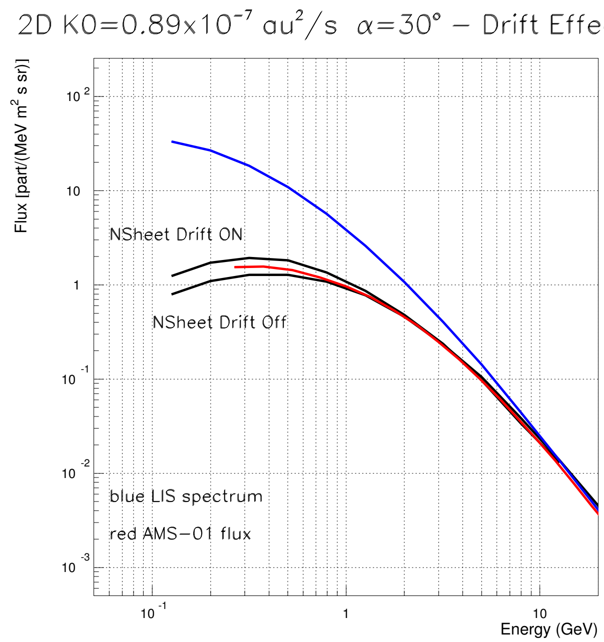

Fig. 3. 2D model: NS drift effect on the modulated CR flux

of ecliptic solar wind velocity V_0 (a higher value of V means higher solar activity).

TABLE I
MODULATION PARAMETERS EFFECT ON SOLAR SYSTEM PLANETS

	CR flux at **Pluto** part/(MeV m^{2}s sr)		CR flux at **Mars** part/(MeV m^{2}s sr)	
Energy in MeV	$K_0 = 0.89$ 10^{-7} au^{2}s^{-1}	$K_0 = 1.00$ 10^{-7} au^{2}s^{-1}	NS drift OFF	NS drift ON
125.9	3.73	4.62	1.10	1.43
199.5	4.79	5.68	1.43	1.72
316.2	4.90	5.61	1.62	2.00
501.2	4.06	4.51	1.57	1.95
794.3	2.77	3.00	1.28	1.45
1258.9	1.59	1.68	0.87	0.91
1995.3	0.78	0.81	0.50	0.50
3162.3	0.33	0.34	0.25	0.24
5011.9	0.13	0.13	0.10	0.09

D. Drift effects

To study the effect of the drift velocity terms we considered a period when $\alpha = 30^o$ and $V = 400$ kms^{-1} for $A > 0$ (see [15] for $A < 0$). After computing the flux including all the terms we have excluded the drift from the transport equation. The drift effect seems to be more evident in positive periods. We can distinguish three terms involving drift velocities in the heliosphere: gradient, curvature and neutral sheet (NS) drifts. We have studied as each one of the drift terms affects the modulated spectrum and it seems that neutral sheet is the dominant one. (for both solar polarities but stronger for $A > 0$). So we evaluated the effect on the 2D model of the presence of this component of drift. As shown in Table I - right part- the spectrum obtained for Mars without NS drift (for a diffusion coefficient $K_0 = 1.00 \times 10^{-7}$ au^{2}s^{-1}) is lower than that one with NS drift, as expecet for protons in positive solar periods (i.e. northern solar hemisphere magnetic field lines directed outward). The same behaviour happens for the Earth, see Fig. 3, where the diffusion coefficient K_0 is fixed at a value of 0.89×10^{-7} au^{2}s^{-1}. The effect of adding the NS drift for the Earth is similar to increasing the diffusion coefficient value (without NS drift, for example at 1.00×10^{-7} au^{2}s^{-1}), see Fig. 2.

IV. RESULTS

A. Comparison with data

We focused our attention in the period with $A > 0$ and we selected June 1998 as the best period for testing the model results, because the AMS-01 [23] experiment at that time has measured one of the most precise proton spectra. We obtain as best value $(K_\perp)_0 = 0.025$, the same as reported in [20], but the rigidity dependence we have found is different: best spectra are obtained for $K_P(\Re) \propto \Re$ (see [14] for simulation of negative solar field polarity, $A < 0$). We have used for this positive solar period ($A > 0$) the following values: for the solar wind speed $V_0 = 400$ kms^{-1} and for the tilt angle $\alpha = 30^o$ (see [24]). AMS spectrum has been used as reference data set for optimize the model. We considered the discrepancy between AMS-01 data and IMP8 data (see [21]) as the uncertainty we have to apply to the model if used for different periods, when only IMP-8 data are available.

TABLE II

SOLAR SYSTEM PLANETS

Planet	Distance (in AU) from the Sun	Space Missions
Mercury	0.387	BepiColombo
Venus	0.723	Venus Express
Earth	1.0	
Mars	1.524	Mars Express
Jupiter	5.203	Ulysses
Saturn	9.539	Cassini-Huygens
Uranus	19.18	
Neptune	30.06	
Pluto	39.44	

TABLE III

MODULATED CR FLUX (PART/(MEV M^2 S SR)) FOR SOLAR SYSTEM PLANETS $K_0 = 0.89 \times 10^{-7}$ AU^{2}S^{-1} (IN PARENTESIS % WITH RESPECT TO THE BURGER LIS SPECTRUM [16])

Planets	Energy in MeV			
	199.5	501.2	1258.9	3162.3
Venus	1.01 (3.77)	1.22	0.73	0.22 (57.5)
Earth	1.09 (4.07)	1.28	0.77	0.23 (57.5)
Mars	1.10 (4.11)	1.28	0.76	0.23 (57.5)
Jupiter	1.15 (4.29)	1.35	0.79	0.23 (57.5)
Saturn	1.31 (4.89)	1.53	0.86	0.24 (60)
Uranus	1.97 (7.36)	2.14	1.07	0.27 (67.5)
Neptune	3.18 (11.88)	3.05	1.33	0.30 (75)
Pluto	4.79 (17.90)	4.06	1.59	0.33 (82.5)
LIS spectrum	26.75	10.90	2.60	0.40

B. CR flux modulation for solar system planets

Here we present our results for the modulated CR spectrum at different heliocentric distances. In particular we focused on the solar system planets, some of which have been already interested in space missions, the others will be investigated by robots or human missions in the near future.

In Table I we already introduced both the effect of the diffusion tensor K_0 and of the NS drift in relation to the heliospheric distance (in this particular case we show the CR flux at Pluto distance, i.e. 39.44 AU from the Sun, and at Mars distance, i.e. 1.52 AU from the Sun).

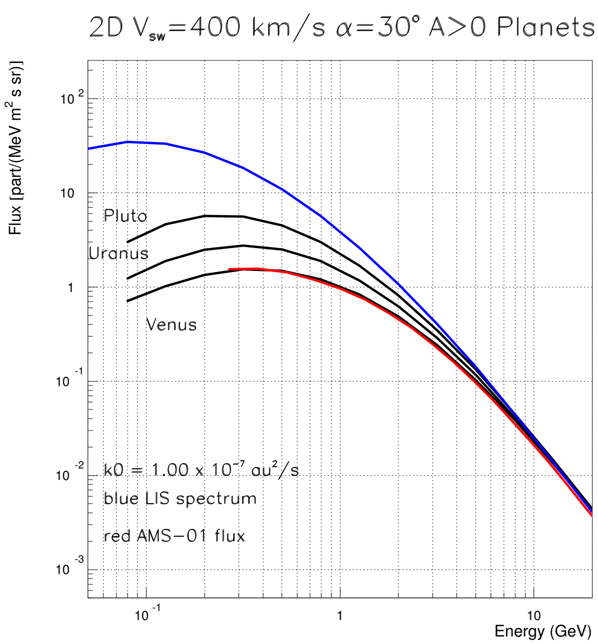

Fig. 4. Modulated CR flux for 3 planets of the Solar System

As shown in Fig. 4 and 5 the solar modulation is decreasing with increasing heliospheric distance (this is in agreement with integral spectrum of CR for example from Voyager I and II and Pioneer, see [25]). Fig. 5 shows the primary CR flux for middle modulation strenght $K_0 = 0.89 \times 10^{-7}$ au^{2}s^{-1} in 3 selected energy bins below 10 GeV: the modulation effect increases for lower energies. (see Table III for all planets and four different energy bins). It is clearly seen that above few GeV the difference between modulated spectra for different planets and the LIS becomes negligible. In Table I is also shown that above $\simeq$ GeV the modulated spectra do not have any strong dependence on the diffusion coefficient (see [**?**].

Fig. 4 shows the CR spectra for Venus, Uranus and Pluto for lower modulation strenght ($K_0 = 1.00 \times 10^{-7}$ au^{2}s^{-1}). As it can be seen comparing Table I and Fig.2 the effect of different modulation strenght (different K_0) is more important for inner planets -Earth- than for outer ones -Pluto- (see Table II for the solar distancies).

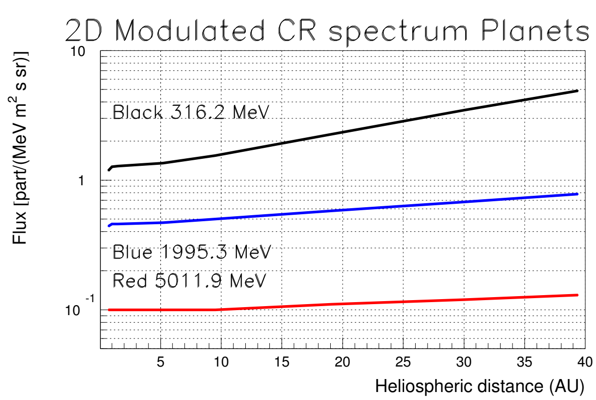

Fig. 5. CR flux modulation for $K_0 = 0.89 \times 10^{-7}$ au^{2}s^{-1} as a function of heliospheric distance

V. CONCLUSIONS

We built a 2D stochastic model (radius and heliolatitude) of particles propagation across the heliosphere starting from a 1D model (only radius). Our model takes into account drift effects and show quantitatively good agreement with measured values. Proton spectra, as predicted by the model, are decreasing with increasing tilt angles and solar wind velocity. We investigated the effect of the different drift componets, in particular the neutral sheet drift. For positive periods as expected the effect of this drift component is to increase the observable flux at 1 AU (Earth). We used a tipical value for $(K_\perp)_0$, the ratio among the parallel and the perpendicular diffusion coefficient. It should be in the range $0.01 - 0.05$ and we choosed 0.025 for our simulations. For this value we could reproduce the spectrum measured by AMS-01 with the following parametrs $V = 400$ kms^{-1} and $\alpha = 30^o$. Diffusion coefficient K_0 disagree with the 1D model predictions. This may be due to the superposition of the latitudinal diffusion and the drift effect (both absent in the 1D model). The model must be tested with data in different solar conditions (in particular $A < 0$ and $\alpha \neq 30^o$), we thus expect the next AMS-02 mission for the long data taking period, more than 3 years, in ascending solar phase and negative polarity. We implemented in the 2D model the possibility to calculate a modulated proton spectra at different heliospheric distancies (corrispondent to the solar system planets radii) and we evaluated for the same positive period of AMS-01 data taking the primary CR flux at different planets. Finally we are able to estimate and predict (thanks to the cyclical solar activity) the proton flux for different space missions scheduled in the near future.

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