Position estimation in monolithic scintillation cameras using B-spline parametrization

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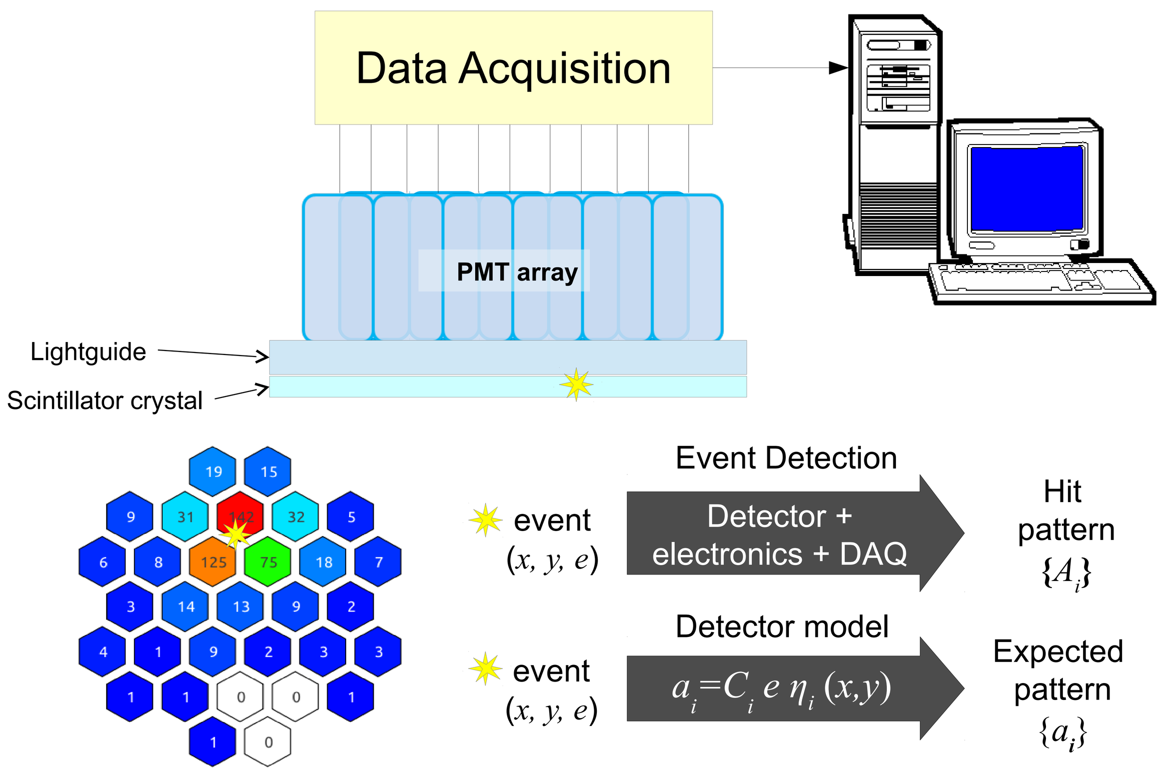
Statistical event reconstruction in a monolithic scintillation camera is an advanced technique which can give better results than traditionally used centre of gravity method. However, to successfully employ this technique, detailed knowledge of PMT light response functions (LRFs) is required. It is also necessary to parametrize the LRFs in order to efficiently store them and use in computer algorithms. In this work advantages of B-spline parametrization are demonstrated on concrete examples of scintillation cameras of different geometry. A software library for fitting simulated and measured light response with spline functions was developed and integrated as a module into an open source software package for simulation and data processing of Anger-type cameras ANTS-II, being developed at LIP-Coimbra.

Gamma camera and Statistical event reconstruction



To reconstruct an event from a hit pattern, find x, y and e for which the expected pattern $\{a_i\}$ is in the best agreement with hit pattern $\{A_i\}$.

How to find this best match?

- Maximum likelihood (Gray & Macovsky, 1976)
- Least squares

Light Response Function $\eta_i(x,y)$ needed

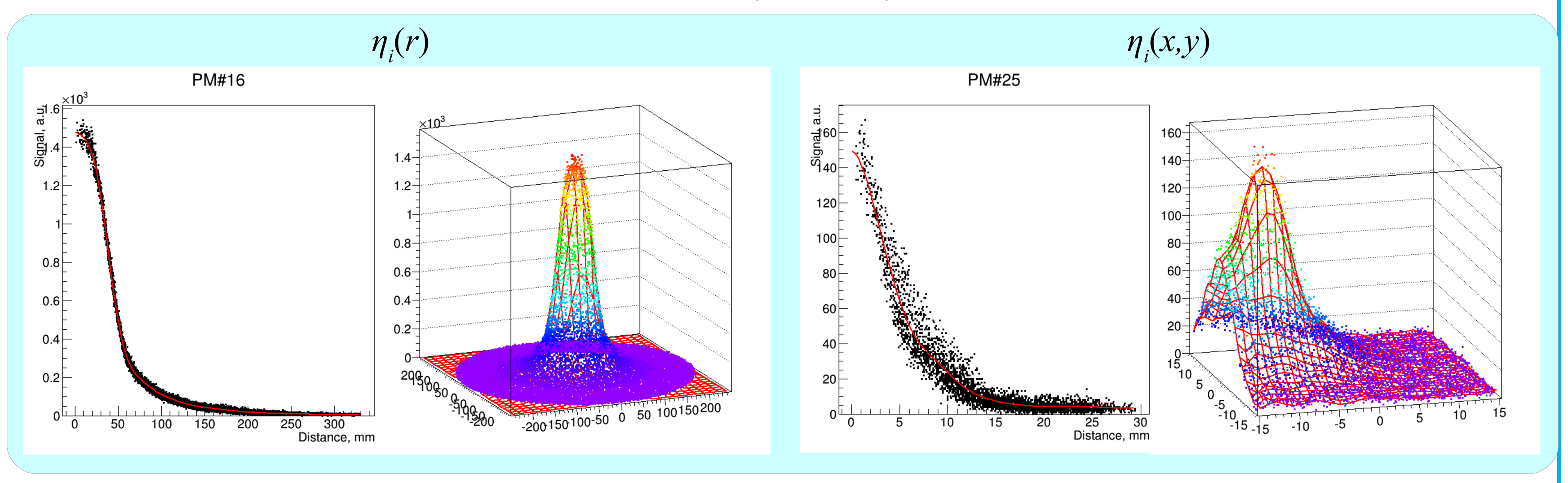
Light Response Functions (LRF)

Light Response Function (LRF) $\eta_i(x,y,z)$ characterizes response of a PMT as a function of a light source position (x,y,z). Depending on the detector geometry, the effective dimensionality of an LRF can be reduced:

1) Thin crystal, little reflections: axially symmetric function $\eta_i(x,y,z) \rightarrow \eta_i(r)$ where r is the distance from the PMT axis

2) Thin crystal, more reflections: 2D function $\eta_i(x,y,z) \rightarrow \eta_i(x,y)$

3) Thick crystal, little reflections: Axial with z-dependence $\eta_i(x,y,z) \rightarrow \eta_i(r,z)$



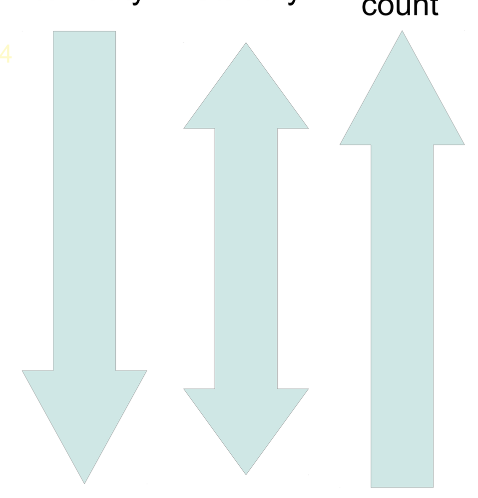
How to find LRFs?

- **Measure** (time consuming in 2D, extremely difficult in 3D)
- **Simulate** (requires detailed knowledge of detector geometry, material optical properties and PMT properties)
- Use **Iterative Reconstruction** with experimental data (a technique first developed for ZEPLIN-III dark matter detector)
  1) Chose a 1st approximation for LRFs (e.g. from simulation)
  2) Reconstruct the event positions using the LRFs
  3) Use the reconstructed event positions to update the LRFs
  4) GOTO 2

Under right conditions, the LRFs converge to the true PMT response.
To do iterative reconstruction efficiently one needs to **parametrize** the LRFs in some way.

LRF Parametrization

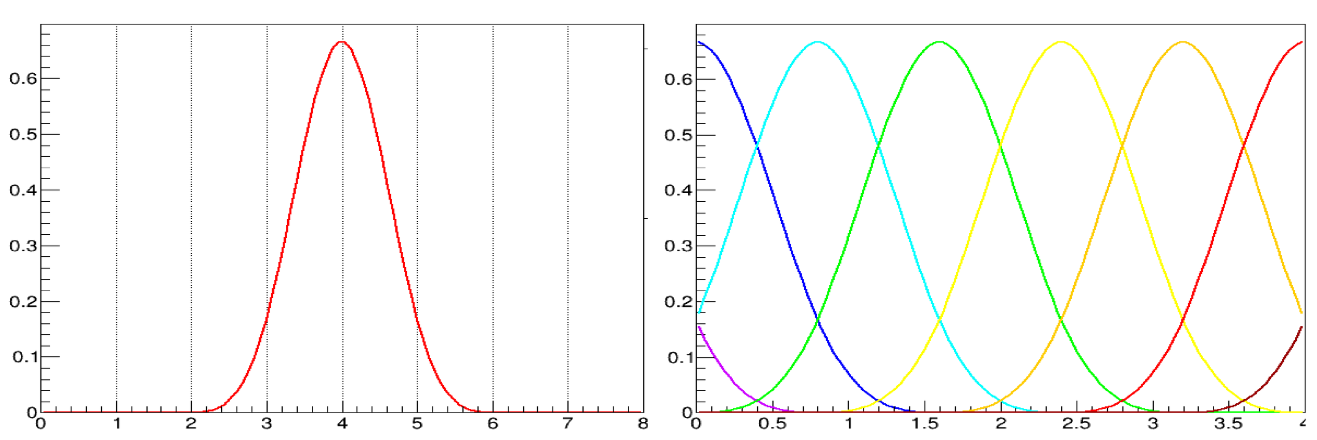
- **Functional** representation
  - More robust than others: fewer parameters and based on detector model
  - Non generic: good representation is geometry-specific and can be tricky to find
  - Typically, non-linear fit
- **Polynomial** representation
  - Generic while easy to implement
  - Linear fit
  - Prone to over-fitting
- **Spline** representation
  - Generic
  - Linear fit
  - More control over fit parameters due to local support
  - Straightforward expansion to 2D and 3D



B-Spline Representation

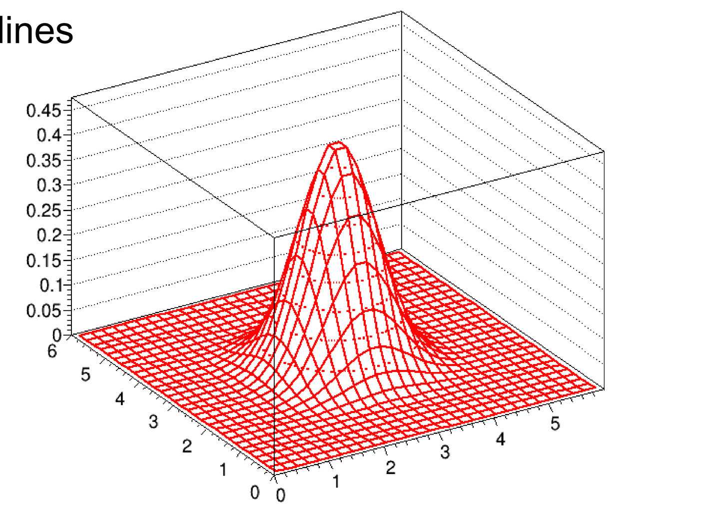
Cubic Uniform Basis Splines (CUBS)

- Smooth 1-st derivative
- Uniform: equidistant knots
- Each CUBS consists of four cubic segments
- Compact support – zero everywhere else
- Basis spline: any cubic spline is a linear combination of B-splines



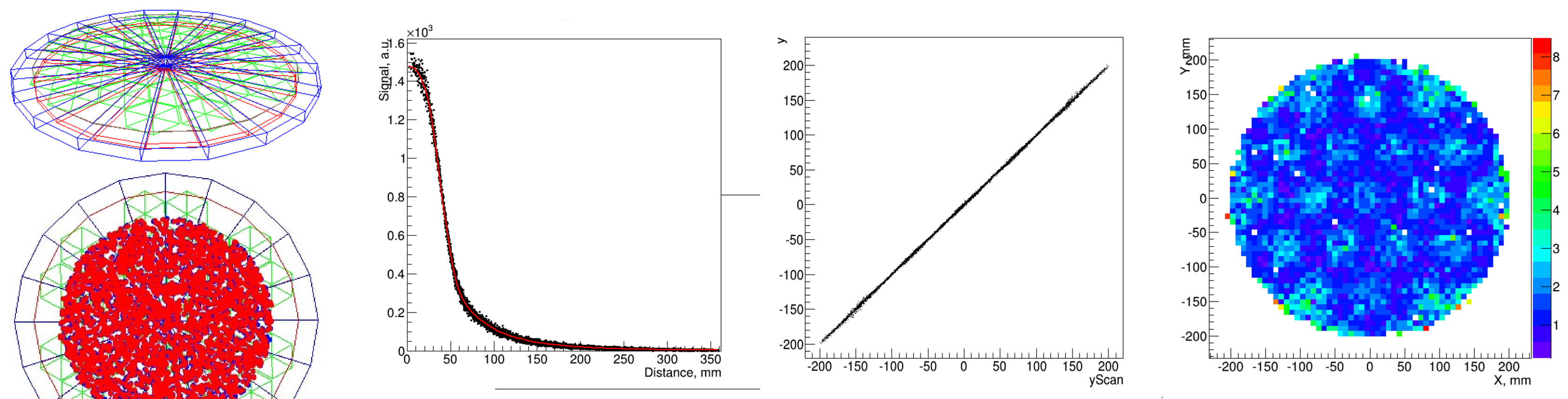
2D: Tensor product splines

- Products of 1D B-splines in both dimendions: $T_{ij}(x,y) = B_i(x)*B_j(y)$
- Consists of 16 bi-cubic segments
- Retains attractive features of B-splines
  - Smoothness
  - Compact support



Pure axial LRFs

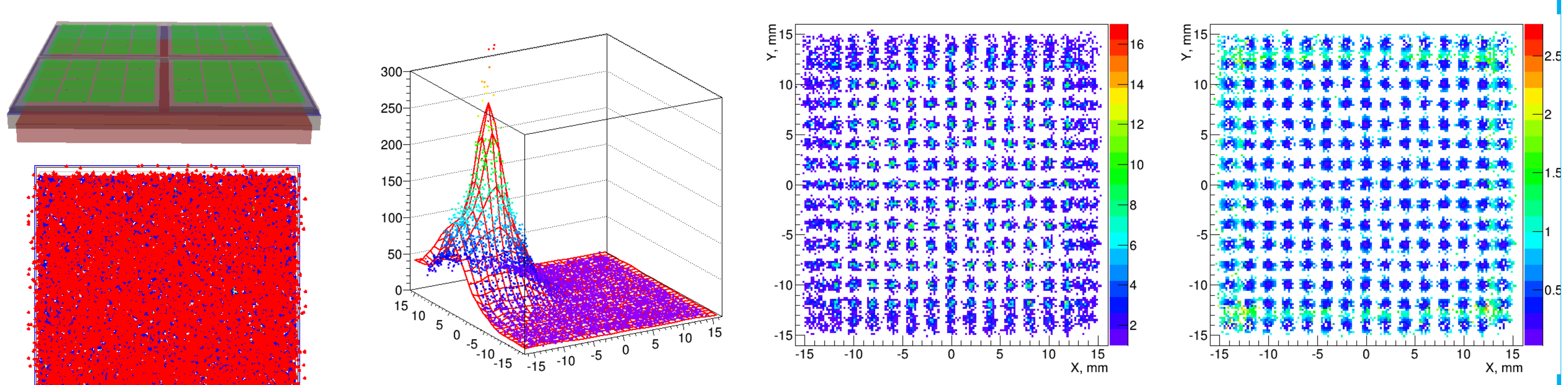
Using a model of commercial gamma camera: NaI crystal Ø 500 mm viewed by 31 PMT. Within the Ø 400 mm field of view (limited by the collimator) axial LRFs permit image reconstruction with practically no distortion



An example of an axially symmetric LRF with B-spline parametrization | Reconstructed y position vs the true one | Average deviation from the true event position across the field of view

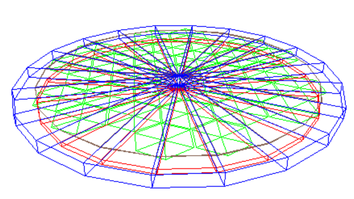
Free-form LRFs

In this model of a compact gamma camera a 30x30x2 mm LSO crystal is coupled through a 1.5 mm thick lightguide to 8x8 SiPM array. The light response considerably deviates from axial symmetry, especially for peripheral SiPMs. Using tensor product spline parametrization it is possible to maintain uniform spatial resolution for the most part of the crystal area.

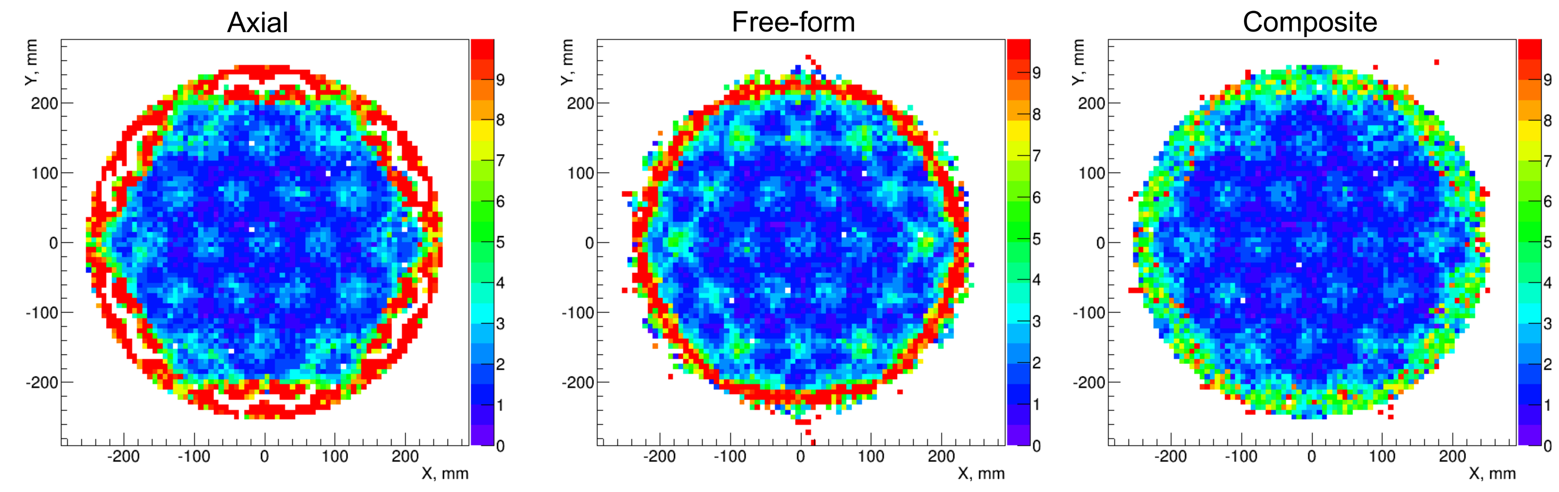


An example of a SiPM light response parametrized as a sum of tensor product splines | Reconstructed event density for simulated 122 keV photocapture on 2x2 mm grid | Average deviation from the true event position across the field of view

Composite LRFs



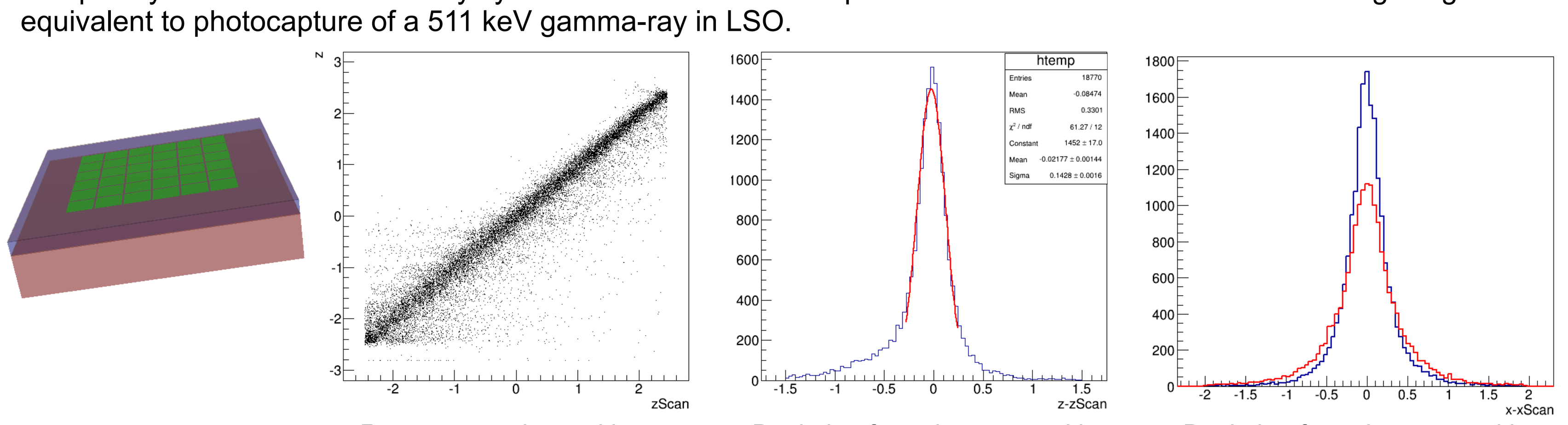
If one wants to expand the field of view of the model of commercial camera to the whole crystal area, than using either axial or free-form LRF will result in distorted image beyond R=200 mm. To solve this problem a composite LRF is used, which is done by fitting the light response with axially symmetric function, calculating the difference between the data and the fit and fitting the difference again with tensor product spline.



Average deviation from the true event position across the extended field of view (Ø 500 mm)

3D reconstruction with axial+Z LRFs

This is a model of compact scintillation camera with depth of interaction sensitivity. It is made of a 30x30x5 mm LSO crystal coupled through a 1.5 mm thick lightguide to 6x6 SiPM array. In this geometry, SiPM response can be adequately modeled with an axially symmetric function with z dependence. The simulation was done for light signals equivalent to photocapture of a 511 keV gamma-ray in LSO.



Reconstructed z position vs the true one | Deviation from the true position in z is 0.35 mm FWHM | Deviation from the true position in x. Red line – axial LRFs, blue line – axial+Z LRFs

ANTS-II LRF module

A software library for fitting simulated and measured light response with spline functions was developed at LIP-Coimbra. It was later integrated as a module into software package for simulation and data processing of Anger-type cameras ANTS-II, also being developed at LIP. In addition to what is described above, the module provides possibility to

- adjust number of knots in the spline representation;
- take into account symmetry of the photosensor array and use a common LRF for a group of sensors thus increasing available statistics;
- compress the tails of axially symmetric LRFs thus improving stability and reducing storage requirements
- visualize LRFs together with the light response data

The ANTS-II software is open source and available at http://coimbra.lip.pt/ants/

Acknowledgements

This work was carried out with financial support from Fundação para a Ciência e Tecnologia (FCT) through the project-grant PTDC/BBB-BMD/2395/2012 (co-financed with FEDER) and from QREN in the frame of the project Rad4Life.